نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۱ کلاس ..... A .....

$$\frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha = |\cos\alpha| \rightarrow \cos\alpha > 0 \rightsquigarrow \text{ربع ۱ و ۴}$$

$$\cot\alpha = \frac{\cos\alpha\ (+)}{\sqrt{1-\cos^2\alpha}\ (+)} \rightarrow \cot\alpha > 0 \rightsquigarrow \text{ربع ۱ و ۳}$$

$$\cos\alpha > 0 \rightsquigarrow$$

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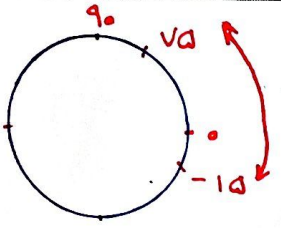

$\frac{R}{12} = 15^\circ$ $\quad -15 < n < 75$

بیش‌ترین و کم‌ترین مقدار $\sin 2n$ را حساب می‌کنیم

if $n = 45^\circ \Rightarrow \sin 2n = \sin 90^\circ = 1$ max

if $n = -15^\circ \Rightarrow \sin 2n = \sin -30^\circ = -\frac{1}{2}$ min

$$-\frac{1}{2} \leq \frac{m-1}{2} \leq 1 \rightarrow -2 < m-1 \leq 2 \Rightarrow -1 < m \leq 3$$

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$$\tan\alpha + \cot\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha} = \frac{1}{\frac{1}{2}\sin 2\alpha} = -3$$

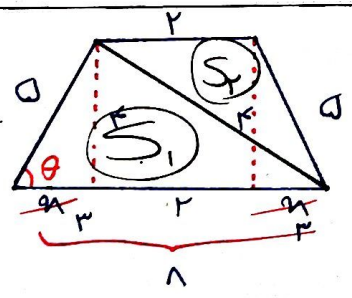
$$\Rightarrow \sin 2\alpha = -\frac{2}{3} \rightarrow \sin\alpha\cos\alpha = -\frac{1}{3}$$

$$(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = 1 - \frac{2}{3} = \frac{1}{3} \rightarrow \sin\alpha + \cos\alpha = \frac{\sqrt{3}}{3}$$

$$(\sin\alpha + \cos\alpha)(\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha)$$

$$\Rightarrow \sin^3\alpha + \cos^3\alpha = \frac{\sqrt{3}}{3} \times \frac{4}{3} = \frac{4\sqrt{3}}{9} \rightarrow -\frac{1}{\sin^3\alpha + \cos^3\alpha} = -\frac{9}{4\sqrt{3}} = -\frac{3\sqrt{3}}{4}$$

۳



$$\cos\theta = \frac{6}{10} = \frac{n}{5} \Rightarrow n = 3 \Rightarrow \sin\theta = \frac{4}{5}$$

$$S_1 = \frac{1}{2}\sin\theta \times 5 \times 8 = \frac{1}{2} \times \frac{4}{5} \times 5 \times 8 = 16$$

$$S_2 = \frac{1}{2}\sin\theta' \times 5 \times 2 = \frac{1}{2} \times \frac{4}{5} \times 5 \times 2 = 4$$

$$S_{ABCD} = 16 + 4 = 20$$

$\theta + \theta' = 180 \Rightarrow \sin\theta = \sin\theta'$

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$$\tan(270 + 15)\tan(180 + 15) - \sin(15)\cos(270 - 15)$$

$$-\cot 15 \times \tan 15 \quad - \quad \sin 15 \times -\sin 15$$

$$-1 + \sin^2 15 = -\cos^2 15 \quad K\cos^2 15 \rightarrow K = -1$$

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$\sqrt{۳}\cos(۲۱۰)\sin(۲۷۰-۲۷) - \sqrt{۲}\sin(۱۳۵)\cos(۱۸۰-۲۷)$

$\sqrt{۳}\times -\frac{\sqrt{۳}}{۲}\times -\cos ۲۷^\circ$ $\qquad -\sqrt{۲}\times\frac{\sqrt{۲}}{۲}\times -\cos ۲۷$

$\frac{۳}{۲}\cos ۲۷^\circ + \cos ۲۷ = \frac{۵}{۲}\cos ۲۷ \rightarrow$ برابر $\frac{۵}{۲}$

۲

$\frac{۱۸۰}{۲۹} = ۵^\circ$ $\qquad ۱۶\cos^۲(۱۵)\cos^۲(۳۰)\cos^۲(۹۰)\cos^۲(۱۲۰)$

$\frac{۳}{۴}\cos^۲ ۱۵^\circ$ $\qquad \frac{۳}{۴}$ $\quad \frac{۱}{۴}$ $\quad \frac{۱}{۴}$

$\cos^۲ ۱۵ = \frac{۱+\cos ۳۰}{۲} = \frac{۱+\frac{\sqrt{۳}}{۲}}{۲} = \frac{۲+\sqrt{۳}}{۴} \Rightarrow f(۹) = \frac{۳}{۴}\times\frac{۲+\sqrt{۳}}{۴} = \frac{۶+۳\sqrt{۳}}{۱۶}$

۸

$\frac{۱-\sin\alpha}{۱+\sin\alpha} = ۲ \rightarrow ۲+۲\sin\alpha = ۱-\sin\alpha \rightarrow ۵\sin\alpha = -۳ \rightarrow \sin\alpha = -\frac{۳}{۵}$

$۱+\cot^۲\alpha = \frac{۱}{\sin^۲\alpha} \Rightarrow \cot\alpha = \frac{۴}{۳} \rightarrow \tan = \frac{۳}{۴}$ $\qquad \tan\frac{\alpha}{۲} = y$ فرض

$\tan\alpha = \frac{۲\tan\frac{\alpha}{۲}}{۱-\tan^۲\frac{\alpha}{۲}} \rightarrow \frac{۳}{۴} = \frac{۲y}{۱-y^۲} \rightarrow ۸y = ۳-۳y^۲ \rightarrow ۳y^۲+۸y-۳=۰$

$y^۲+۸y-۹=۰ \rightarrow (y-۱)(y+۹) \longrightarrow$ $y_۱ = \frac{۱}{۳}$ قابل قبول $y_۲ = -۳$ $\longrightarrow \tan\frac{\alpha}{۲} = \frac{۱}{۳}$ (رد)

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$\frac{\sin\alpha}{۱+\cos\alpha} = \frac{۲\sin\frac{\alpha}{۲}\cos\frac{\alpha}{۲}}{۲\cos^۲\frac{\alpha}{۲}} = \tan\frac{\alpha}{۲} \Rightarrow \frac{۱+\cos\alpha}{\sin\alpha} = \cot\frac{\alpha}{۲}$

$\frac{۱-\cos\alpha}{\sin\alpha} = \frac{۲\sin^۲\frac{\alpha}{۲}}{۲\sin\frac{\alpha}{۲}\cos\frac{\alpha}{۲}} = \tan\frac{\alpha}{۲} \Rightarrow \frac{\sin\alpha}{۱-\cos\alpha} = \cot\frac{\alpha}{۲}$

$+ \Big\} ۲\cot\frac{\alpha}{۲}$

$K = ۲$

۱۰

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(\frac{۳\pi}{۴}+\alpha\right) = \cos\frac{۳\pi}{۴}\times\cos\alpha - \sin\frac{۳\pi}{۴}\times\sin\alpha$

$\cos\alpha = -\sqrt{۱-\sin^۲\alpha} = -\sqrt{\frac{۹۸}{۱۰۰}} = -\frac{۷\sqrt{۲}}{۱۰}$ $\qquad -\frac{\sqrt{۲}}{۲}\times -\frac{۷\sqrt{۲}}{۱۰} - \frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰} = \frac{۷}{۱۰} - \frac{۱}{۱۰}$

$\frac{۳}{۵} \leftarrow \frac{۶}{۱۰}$