رادین مکلیان یازدهم A

۱۹,۷۵

-۱ ۲

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \cos\alpha > 0$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$$

α ناحیه اول است

-۲ ۱,۷۵

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \underbrace{\sin 2x}_{\frac{m-1}{4}} \le 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$\Rightarrow -1 < m \le 5$$ دقت!

-۳ ۲

$$\tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \Rightarrow \sin x \cdot \cos x = -\frac{1}{3}$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{3} \Rightarrow (\sin x + \cos x)^2 = \frac{1}{3}$$

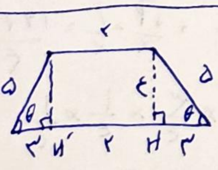
$3\pi < 4x < 4\pi \Rightarrow \frac{3}{4}\pi < x < \pi$, $\sin x + \cos x < 0$

$$\Rightarrow \sin x + \cos x = -\sqrt{\frac{1}{3}} \Rightarrow \sin^3 x + \cos^3 x = (\sin x + \cos x)^3 - 3\sin x\cos x(\sin x + \cos x)$$

$$= -\frac{1}{3}\sqrt{\frac{1}{3}} - 3\left(-\frac{1}{3}\right)\left(-\frac{1}{\sqrt{3}}\right) = -\frac{1}{3}\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{3}} = -\frac{4}{3}\sqrt{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{4}{3}\sqrt{\frac{1}{3}}} = \frac{1}{-\sqrt{\frac{16}{27}}} = -\frac{\sqrt{27}}{\sqrt{16}} = -\frac{3}{4}\sqrt{3}$$

-۴ ۲



$$\overset{\triangle}{ABH}: \cos\theta = \frac{BH}{AB} = \frac{BH}{5} = \frac{6}{10} \Rightarrow BH = 3$$

فیثاغورس $\Rightarrow AH^2 + BH^2 = AB^2 \Rightarrow AH^2 + 9 = 25 \Rightarrow AH^2 = 16 \Rightarrow AH = 4$

$$\left.\begin{aligned} AB &= DC \\ \hat{H}_1 &= \hat{H}'_1 = 90 \\ C &= B = \theta \end{aligned}\right\} \overset{\text{وز}}{\Longrightarrow} \overset{\triangle}{ABH} \cong DCH' \Longrightarrow CH' = BH = 3$$

$CH' + H'H + BH$

$$S = \frac{AD + BC}{2} \times AH = \frac{2+8}{2} \times 4 = 20$$

$$\left.\begin{aligned} &AD \parallel HH' \\ &\left.\begin{aligned} AH &\perp BC \\ DH' &\perp BC \end{aligned}\right\} AH \parallel DH' \\ &\hat{H}_2 = 90 \end{aligned}\right\} \underset{\text{مستطیل}}{ADH'H} \Rightarrow AD = HH' = 2$$

۴۵

$\tan(۲۸۵^\circ)\tan(-۱۶۵) - \sin(۱۰۹۵)\cos(۲۵۵) = \underbrace{-\cot(۱۵)\tan(۱۵)}_{-۱} - \sin(۱۵)(-)(\sin ۱۵)$

۲

$\tan(۲۸۵) = \tan(۲۷۰+۱۵) = \tan(\frac{۳\pi}{۲}+۱۵) = -\cot ۱۵$

$\tan(-۱۶۵) = \tan(۱۵-۱۸۰) = \tan(۱۵-\pi) = \tan ۱۵$

$\sin(۱۰۹۵) = \sin(۱۰۸۰+۱۵) = \sin(۶\pi+۱۵) = \sin ۱۵$

$\cos(۲۵۵) = \cos(۲۷۰-۱۵) = \cos(\frac{۳\pi}{۲}-۱۵) = -\sin ۱۵$ $\Rightarrow$

$-۱+\sin^۲(۱۵) = -(۱-\sin^۲ ۱۵) = -\cos^۲ ۱۵ = k\cos^۲(۱۵) \Rightarrow k = -۱$ ✓

۴۶

$A = \sqrt{۳}\cos(۲۱۰)\underbrace{\sin(۲۴۳)}_{-\cos(۲۷)} - \sqrt{۲}\sin(۱۳۵)\underbrace{\cos(۱۵۳)}_{-\cos(۲۷)}$

۲

$= \sqrt{۳}(-\frac{\sqrt{۳}}{۲})(-)\cos(۲۷) - \sqrt{۲}(\frac{\sqrt{۲}}{۲})(-)\cos(۲۷) = \frac{۳}{۲}\cos(۲۷) + \cos(۲۷) = \frac{۵}{۲}\cos(۲۷)$

$\frac{\frac{۵}{۲}\cos(۲۷)}{\cos(۲۷)} = \frac{۵}{۲}$ ✓

۴۷

$f(\frac{\pi}{۲۴}) = ۱۶\cos^۲(\frac{\pi}{۱۲}) \times \cos^۲(\frac{\pi}{۶})\cos^۲(\frac{\pi}{۳})\cos^۲(\frac{۲\pi}{۳})$

۲

$\cos^۲(\frac{۲\pi}{۳}) = -\frac{۱}{۲}\times -\frac{۱}{۲} = \frac{۱}{۴}$, $\cos^۲(\frac{\pi}{۳}) = \frac{۱}{۲}\times\frac{۱}{۲} = \frac{۱}{۴}$

$\cos^۲(\frac{\pi}{۶}) = \frac{\sqrt{۳}}{۲}\times\frac{\sqrt{۳}}{۲} = \frac{۳}{۴}$ $\quad \cos^۲(\frac{\pi}{۱۲}) = \cos^۲(۱۵) = \cos^۲(۴۵-۳۰)$

$\cos(۴۵-۳۰) = \cos(۴۵)\cos(۳۰) + \sin(۴۵)\sin(۳۰) \Rightarrow \cos(۴۵-۳۰) = \frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۳}}{۲} + \frac{\sqrt{۲}}{۲}\times\frac{۱}{۲} = \frac{\sqrt{۶}+\sqrt{۲}}{۴}$

$\Rightarrow \cos^۲(۱۵) = \frac{۶+۲+۲\sqrt{۱۲}}{۴\times۴} = \frac{۲+\sqrt{۳}}{۴}$ $\Rightarrow f(x) = ۱۶\times\frac{۲+\sqrt{۳}}{۴}\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴}$

$= \frac{۳\times(۲+\sqrt{۳})}{۴\times۴} = \frac{۶+۳\sqrt{۳}}{۱۶}$ ✓

۴۸

$۱-\sin x = ۴+۴\sin x \Rightarrow ۵\sin x = -۳ \Rightarrow \sin x = -\frac{۳}{۵} \Rightarrow \sin^۲ x = \frac{۹}{۲۵} \Rightarrow ۱-\sin^۲ = \frac{۱۶}{۲۵}$

۲

$\Rightarrow \cos x = \frac{۴}{۵}$

$\tan\frac{x}{۲} = \frac{\sin x}{۱+\cos x} = \frac{-\frac{۳}{۵}}{۱-\frac{۴}{۵}} = -۳$ ✓

۳

$$\frac{\sin\theta}{1-\cos\theta}=\frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=\cot\frac{\theta}{2}$$

$$\frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}\Rightarrow\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}=\cot\frac{\theta}{2}$$

$$\Rightarrow\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\cot\left(\frac{\theta}{2}\right)=k\cot\frac{\theta}{2}\Rightarrow k=2\ \checkmark$$

$$\sin\alpha=\frac{\sqrt{2}}{10}\Rightarrow\sin^2\alpha=\frac{2}{10}\Rightarrow 1-\sin^2\alpha=\frac{98}{100}\Rightarrow\cos^2\alpha=\frac{98}{100}\Rightarrow\cos\alpha=\pm\frac{7\sqrt{2}}{10}$$ ۱۱۰

۳

$$\xrightarrow{\alpha\ \text{ربع دوم}}\cos\alpha=\frac{-7\sqrt{2}}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(\frac{11\pi}{4}\right)\times\cos(\alpha)-\sin\left(\frac{11\pi}{4}\right)\sin(\alpha)=\left(-\frac{\sqrt{2}}{2}\times\frac{-7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}\right)$$

$$=\frac{14}{20}-\frac{2}{10}=\frac{12}{20}=\frac{6}{10}\ \checkmark$$