نام و نام خانوادگی ...مهدی ابوالفضلی... پاسخنامه تشریحی تکلیف شماره ...۲۷... کلاس ...یازدهم ۱۱ ریاضی A...

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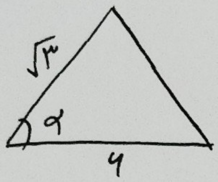
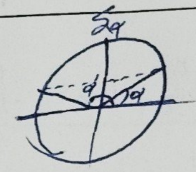


$S = 4.5 \qquad \alpha + \alpha' = 180 \xrightarrow{\text{مکمل اند}} S\alpha = S\alpha'$



$\frac{\alpha_{max}}{\alpha_{min}} = ?$

$S_{\text{مثلث}} = \frac{1}{2} ab \sin\alpha = \frac{1}{2} ab\, S\alpha' \longrightarrow \cancel{\frac{1}{2}} \times \sqrt{3} \times 6 \times S\alpha = \frac{9}{\cancel{2}} \Rightarrow S\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow S\alpha = \frac{\sqrt{3}}{2} \begin{cases} \alpha = 60 \longrightarrow min \\ \alpha' = 120 \longrightarrow max \end{cases} \Rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{120}{60} = 2$

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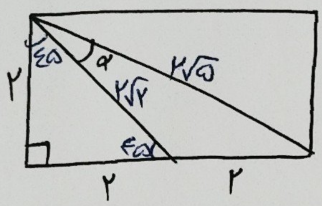
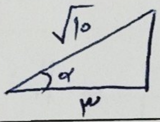


$Cot\alpha = ?$

$C = \sqrt{a^2+b^2-2ab\,C\alpha} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{5})^2 - 2(\underbrace{2\sqrt{5}\cdot 2\sqrt{2}}_{4\sqrt{10}})\cdot Cos\alpha} = 2$

$\Rightarrow C = 2 = \sqrt{8 + 20 - 8\sqrt{10}\, Cos\alpha} = \sqrt{4} \Rightarrow 28 - 8\sqrt{10}\, Cos\alpha = 4$

$\Rightarrow 8\sqrt{10}\, Cos\alpha = 24 \xrightarrow{3} Cos\alpha = \frac{3}{\sqrt{10}}$



$\Rightarrow Cot\alpha = 3$

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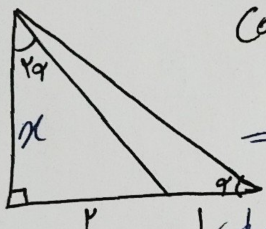

$Cot\alpha = ? \longrightarrow Cot 2\alpha = \frac{1-\tan^2\alpha}{2\tan\alpha}$

$\Rightarrow \tan\alpha = \frac{x}{3} \longrightarrow Cot 2\alpha = \frac{1 - \frac{x^2}{9}}{\frac{2x}{3}} = \frac{x}{2}$

$\xrightarrow{\text{طرفین وسطین}} \frac{2x^2}{6} = \frac{x^2}{3} = 1 - \frac{x^2}{9} \longrightarrow \frac{4x^2}{9} = 1$

$\longrightarrow \frac{3x^2 + x^2}{9} = 1$

$\Rightarrow x = \frac{3}{2}$

$\Rightarrow Cot\alpha = \frac{3}{x} = \frac{3}{1.5} = 2$

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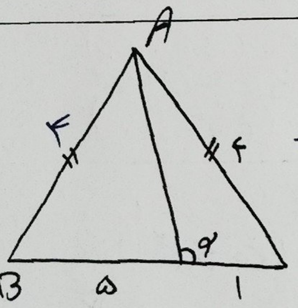
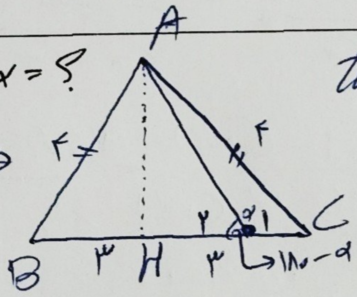


$\tan\alpha = ?$

$\Rightarrow$

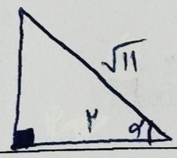


$\tan\alpha = -\tan(180-\alpha)$

$AH = \sqrt{16-9} = \sqrt{7}$

$\longrightarrow \alpha \longrightarrow$ ربع دوم

$\tan\alpha \longrightarrow \sqrt{7}$



$\Rightarrow |\tan\alpha| = \frac{\sqrt{7}}{2} \Rightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$

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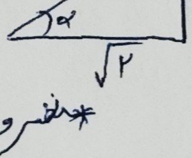
$2\sin^2\alpha + Cos^2\alpha = \frac{4}{3} \longrightarrow S^2\alpha + \underbrace{S^2\alpha + Cos^2\alpha}_{1} = \frac{4}{3} \longrightarrow S^2\alpha = \frac{1}{3} \Rightarrow S\alpha = \frac{1}{\sqrt{3}}$

$\tan^2\alpha = ? \longrightarrow S\alpha = \frac{1}{\sqrt{3}}$



$\Rightarrow \tan\alpha = \frac{1}{\sqrt{2}} \longrightarrow \tan^2\alpha = \frac{1}{2}$

* منفی و مثبت بودن آن مهم نیست چون توان ۲ است!

۶

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=?$$

$$\Rightarrow \sin^4\alpha=(\sin^2\alpha)^2=(1-\cos^2\alpha)^2=\cos^4-2\cos^2+1$$

$$\cos^4\alpha=(\cos^2)^2=(1-\sin^2\alpha)^2=\sin^4\alpha-2\sin^2\alpha+1$$

$$\to \frac{\cos^4-2\cos^2\alpha+1+4\cos^2\alpha}{1+\cos^2}-\frac{\sin^4\alpha-2\sin^2\alpha+1+4\sin^2}{1+\sin^2\alpha}=\frac{(\cos^2\alpha+1)^{\cancel{2}}}{\cancel{\cos^2+1}}-\frac{(\sin^2+1)^{\cancel{2}}}{\cancel{\sin^2+1}}$$

$$\Rightarrow \cos^2\alpha+\cancel{1}-\sin^2-\cancel{1}=\cos^2-\sin^2\alpha=\cos2\alpha$$

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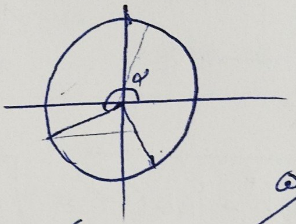
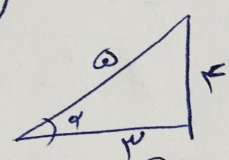
$\tan\alpha=\frac{4}{3} \longrightarrow \alpha \longrightarrow$ ربع سوم



$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)=?$$

$$+\cos\alpha\ \cdot\ -\sin\alpha\ \ -\ \ -\cot\alpha$$

$$\Rightarrow -\sin\alpha\cdot\cos\alpha+\cot\alpha \qquad \Rightarrow \tan\alpha=\frac{4}{3}\longrightarrow$$



$$\Rightarrow \frac{4}{5}\times\frac{-3}{5}+\frac{3}{4}\longrightarrow\frac{-12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\frac{27}{100}=0/27$$

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$$(3\cos4\alpha+\sqrt{2}\sin\alpha-\sqrt{2}\cos\alpha)$$

$$\frac{\pi}{12}=15^\circ\longrightarrow\sin15=\cos75 \longrightarrow 75=(30+45)$$

$$\alpha=\frac{\pi}{12}\longrightarrow 4\cos\overset{60}{\frac{\pi}{3}}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}$$

$$\Rightarrow \sin75=\sin(30+45)=\sin30\cdot\cos45+\cos30\sin45=\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\cos75=\cos(30+45)=\cos30\cdot\cos45-\sin30\cdot\sin45=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

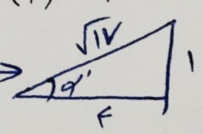
$$\Rightarrow 3\cos60+\sqrt{2}(\cos75-\sin75)=3\times\frac{1}{2}+\sqrt{2}\left(\frac{\cancel{\sqrt{6}}-\sqrt{2}-\sqrt{2}-\cancel{\sqrt{6}}}{4}\right)=\frac{3}{2}+\frac{-2\sqrt{2}\times\sqrt{2}}{4}=\frac{3}{2}-1=\frac{1}{2}$$

۹

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=?=\frac{\frac{\sin\alpha}{\cos\alpha}-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{\sin\alpha-\sin\alpha\cos\alpha}{\cos\alpha}}{\sin\alpha-\cos\alpha}=\frac{\frac{\sin\alpha(1-\cos\alpha)}{\cos\alpha}}{\sin\alpha-\cos\alpha}=\frac{\sin\alpha(1-\cos\alpha)}{\cos\alpha(\sin\alpha-\cos\alpha)}$$

$$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4}\xrightarrow{\text{تغییر متغیر}}\frac{\alpha}{2}=\alpha'\to\frac{\sin2\alpha'(1-\cos2\alpha')}{\cos2\alpha'(\sin2\alpha'-\cos2\alpha')}=\frac{2\sin\alpha'\cos\alpha'(\cancel{1}-(\cos^2\alpha'-\sin^2\alpha'))}{(\cos^2\alpha'-\sin^2\alpha')(2\sin\alpha'\cos\alpha'-(\cos^2\alpha'-\sin^2\alpha'))}$$

$$\sin^2+\cos^2-\cos^2+\sin^2 \qquad -(\cos^2-\sin^2)(\sin^2-\cos^2+2\sin\cos)$$



$$\Rightarrow \frac{2\sin\cos(2\sin^2)}{(\sin^2-\cos^2)\cdot(\sin^2-\cos^2+2\sin\cos)}=\frac{2\times\frac{1\times4}{17}\left(2\cdot\frac{1}{17}\right)}{\left(\frac{1}{17}-\frac{16}{17}\right)\left(\frac{1}{17}-\frac{16}{17}+\frac{8}{17}\right)}=\frac{-\frac{8}{17}\cdot\frac{2}{17}}{\frac{-15}{17}\cdot\frac{-7}{17}}=\frac{\frac{-16}{17\times17}}{\frac{15\times7}{17\times17}}=\frac{-16}{105}$$

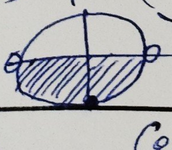
۱۰

$$2\sin\alpha<\sin2\alpha\longrightarrow\sin2\alpha=2\sin\alpha\cdot\cos\alpha$$

$$0<\frac{\cot\alpha}{\sin\alpha} \qquad \Rightarrow \cancel{2}\sin\alpha<\cancel{2}\sin\alpha\cdot\cos\alpha\longrightarrow\sin\alpha<\underbrace{\sin\alpha\cdot\cos\alpha}$$

از آنجایی که این عبارت در $\cos\alpha$ ضرب شده و $-1 \leq \cos\alpha \leq +1$ است و همواره از نظر عددی (بزرگی) کوچکتر می‌شود مگر (آنکه $\cos\alpha=1$ باشد که آنگاه مساوی می‌شود) در نتیجه متوجه می‌شویم که $\sin\alpha$ عددی منفی بوده که باعث بزرگتر شدن آن شده است.

$\alpha\longrightarrow ?$ کدام ربع



$$\frac{\cot\alpha}{\sin\alpha}>0\longrightarrow\cot\alpha\ominus\Rightarrow\frac{\cos\alpha^{+}}{\sin\alpha_{-}}\Rightarrow\alpha:\ \text{ربع ۴}$$

$$\sin\alpha\longrightarrow\ominus$$