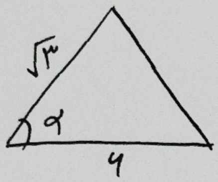
نام و نام خانوادگی ...مهدی ابوالفضلی... پاسخنامه تشریحی تکلیف شماره ...۲۷... کلاس ...یازدهم ۱۱ ریاضی A...

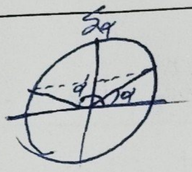
۲۰

۱ (۲)



$S = 4,5$

$\alpha + \alpha' = 180 \xrightarrow{\text{مکمل اند}} S\alpha = S\alpha'$

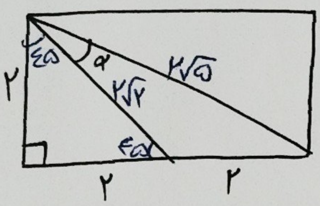


$\dfrac{\alpha\ max}{\alpha\ min} = ?$

$S_{\text{مثلث}} = \frac{1}{2}ab\,Sin\alpha = \frac{1}{2}ab\,S\alpha' \longrightarrow \frac{1}{2} \times \sqrt{3} \times 6 \times S\alpha = \frac{9}{2} \Rightarrow S\alpha = \frac{\sqrt{3}}{2}$

$\Longrightarrow S\alpha = \frac{\sqrt{3}}{2} \begin{cases} \alpha = 60 \longrightarrow min \\ \alpha' = 120 \longrightarrow max \end{cases} \Longrightarrow \dfrac{\alpha\ max}{\alpha\ min} = \dfrac{120}{60} = 2$

۲ (۲)

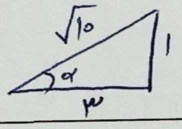


$Cot\alpha = ?$

$C = \sqrt{a^2+b^2-2ab\,C\alpha} = \sqrt{(2\sqrt{2})^2+(2\sqrt{5})^2-2\underbrace{(2\sqrt{5}\cdot 2\sqrt{2})}_{4\sqrt{10}}\cdot Cos\alpha} = 2$

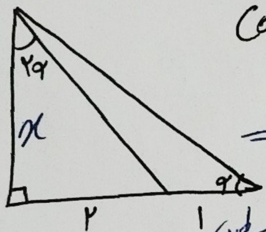
$\Longrightarrow C = 2 = \sqrt{8+20-8\sqrt{10}\,Cos\alpha} = \sqrt{4} \Longrightarrow 28 - 8\sqrt{10}\,Cos\alpha = 4$

$\Longrightarrow 8\sqrt{10}\,Cos\alpha = 24 \longrightarrow Cos\alpha = \frac{3}{\sqrt{10}}$



$\Longrightarrow Cot\alpha = 3$

۳ (۲)



$Cot\alpha = ? \longrightarrow Cot2\alpha = \dfrac{1-tan^2\alpha}{2\,tan\alpha}$

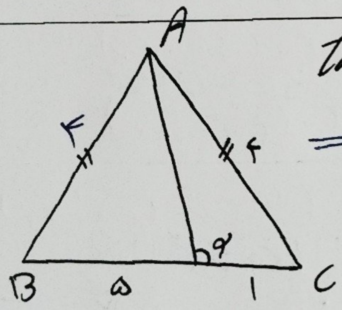
$\Longrightarrow tan\alpha = \frac{x}{3} \longrightarrow Cot2\alpha = \dfrac{1-\frac{x^2}{9}}{\frac{2x}{3}} = \frac{x}{2}$

$\xrightarrow{\text{طرفین وسطین}} \frac{2x^2}{6} = \frac{x^2}{3} = 1 - \frac{x^2}{9} \longrightarrow \frac{3x^2+x^2}{9} = 1 \longrightarrow \frac{4x^2}{9} = 1$

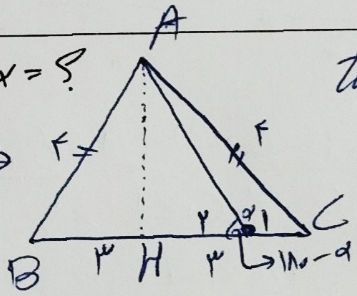
$\Longrightarrow x = \frac{3}{2}$

$\Longrightarrow Cot\alpha = \frac{3}{x} = \frac{3}{1,5} = 2$

۴ (۲)



$tan\alpha = ? \Longrightarrow$

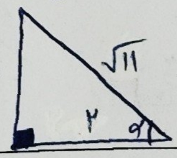


$tan\alpha = -tan(180-\alpha)$

$AH = \sqrt{16-9} = \sqrt{7}$

$\longrightarrow \alpha \longrightarrow$ ربع دوم

$tan\alpha \longrightarrow \sqrt{7}$

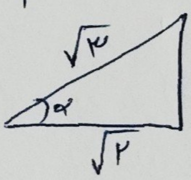


$\Longrightarrow |tan\alpha| = \frac{\sqrt{7}}{2} \Longrightarrow tan\alpha = -\frac{\sqrt{7}}{2}$

۵ (۲)

$2Sin^2\alpha + Cos^2\alpha = \frac{4}{3} \longrightarrow S^2\alpha + \underbrace{S^2\alpha + Cos^2\alpha}_{1} = \frac{4}{3} \longrightarrow S^2\alpha = \frac{1}{3} \Rightarrow S\alpha = \frac{1}{\sqrt{3}}$

$tan^2\alpha = ? \longrightarrow S\alpha = \frac{1}{\sqrt{3}}$



$\Longrightarrow tan\alpha = \frac{1}{\sqrt{2}} \longrightarrow tan^2\alpha = \frac{1}{2}$

*منفی و مثبت بودن آن مهم نیست چون توان ۲ است!

۶ — ۲

$$\frac{S^4\alpha + 4\cos^2\alpha}{1+C^2\alpha} - \frac{C^4\alpha + 4S^2\alpha}{1+S^2\alpha} = ?$$

$S^4\alpha = (S^2\alpha)^2 = (1-\cos^2\alpha)^2 = \cos^4 - 2\cos^2 + 1$

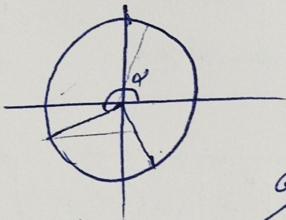
$\cos^4\alpha = (\cos^2)^2 = (1-S^2\alpha)^2 = S^4\alpha - 2S^2\alpha + 1$

$$\longrightarrow \frac{\cos^4 - 2\cos^2\alpha + 1 + 4C^2\alpha}{1+C^2} - \frac{S^4\alpha - 2S^2\alpha + 1 + 4S^2}{1+S^2\alpha} = \frac{(\cos^2\alpha+1)^{\cancel{2}}}{\cancel{C^2+1}} - \frac{(S^2+1)^{\cancel{2}}}{\cancel{S^2+1}}$$

$$\Longrightarrow \cos^2\alpha + \cancel{1} - S^2 - \cancel{1} = C^2 - S^2\alpha = \cos 2\alpha$$

۷ — ۲

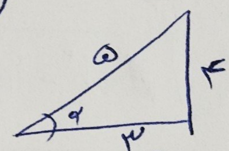
$\tan\alpha = \frac{4}{3} \longrightarrow \alpha \longrightarrow$ ربع سوم



$$S\left(\cancel{\tfrac{9\pi}{2}}+\alpha\right) C\left(\cancel{\tfrac{7\pi}{2}}-\alpha\right) - \tan\left(\alpha - \cancel{\tfrac{3\pi}{2}}\right) = ?$$

$+\cos\alpha \;\cdot\; -S\alpha \;-\; -\cot\alpha$

$\Longrightarrow -S\alpha\cdot C\alpha + \cot\alpha \Longrightarrow \tan\alpha = \frac{4}{3} \longrightarrow$



$$\Longrightarrow \frac{4}{5}\times\frac{-3}{5} + \frac{3}{4} \longrightarrow \frac{-12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100} = 0٫27$$

۸ — ۲

$(3\cos 4\alpha + \sqrt{2}\,S\alpha - \sqrt{2}\,C\alpha)$ $\quad \frac{\pi}{12} = 15^\circ \longrightarrow S15 = \cos 75 \longrightarrow 75 = (30+45)$

$\alpha = \frac{\pi}{12} \longrightarrow 4\cos\cancel{\frac{\pi}{3}}^{60} + \sqrt{2}\,S\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12}$

$\Longrightarrow S75 = S(30+45) = S30\cdot\cos45 + \cos30\,S45 = \frac{1}{2}\times\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

$\cos75 = \cos(30+45) = \cos30\cdot\cos45 - S30\cdot S45 = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$$\Longrightarrow 3\cos60 + \sqrt{2}(\cos75 - S75) = 3\times\frac{1}{2} + \sqrt{2}\left(\frac{\cancel{\sqrt{6}}-\sqrt{2}-\sqrt{2}-\cancel{\sqrt{6}}}{4}\right) = \frac{3}{2} + \cancel{\frac{-2\sqrt{2}\times\sqrt{2}}{4}} = \frac{3}{2} - 1 = \frac{1}{2}$$

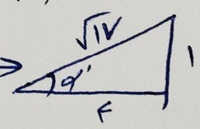
۹ — ۲

$$\frac{\tan\alpha - S\alpha}{S\alpha - C\alpha} = ? = \frac{\frac{S\alpha}{C\alpha} - S\alpha}{S\alpha - \cos\alpha} = \frac{\frac{S\alpha - S\alpha C\alpha}{C\alpha}}{S\alpha - C\alpha} = \frac{\frac{S\alpha(1-C\alpha)}{C\alpha}}{S\alpha - C\alpha} = \frac{S\alpha(1-C\alpha)}{C\alpha(S\alpha - C\alpha)}$$

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4} \xrightarrow{\text{تغییر متغیر}} \left\{\frac{\alpha}{2} = \alpha' \right. \longrightarrow \frac{S2\alpha'(1-C2\alpha')}{C2\alpha'(S2\alpha' - C2\alpha')} = \frac{2S\alpha' C\alpha'\left(\cancel{1}-(C^2\alpha' - S^2\alpha')\right)}{C^2\alpha' - S^2\alpha'\left(2SC\alpha' - (C^2\alpha' - S^2\alpha')\right)}$

$S^2 + C^2 - C^2 + S^2$

$-(C^2 - S^2)(S^2 - C^2 + 2SC)$



$$\Longrightarrow \frac{2SC(2S^2)}{(S^2 - C^2)\cdot(S^2 - C^2 + 2SC)} = \frac{2\times\frac{1\times4}{17}\left(2\cdot\frac{1}{17}\right)}{\left(\frac{1}{17}-\frac{16}{17}\right)\left(\frac{1}{17}-\frac{16}{17}+\frac{8}{17}\right)} = \frac{\frac{-8}{\cancel{17}}\cdot\frac{2}{\cancel{17}}}{\frac{-15}{\cancel{17}}\cdot\frac{-7}{\cancel{17}}} = \frac{\frac{-16}{\cancel{17\cdot17}}}{\frac{15\times7}{\cancel{17\times17}}} = \frac{-16}{105}$$

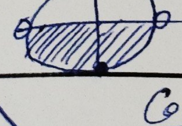
۱۰ — ۲

$2S\alpha < S2\alpha \longrightarrow S2\alpha = 2S\alpha\cdot\cos\alpha$

$0 < \frac{\cot\alpha}{S\alpha}$ $\quad \Longrightarrow \cancel{2}S\alpha < \cancel{2}S\alpha\cdot\cos\alpha \longrightarrow S\alpha < \underbrace{S\alpha\cdot\cos\alpha}$

از آنجایی که این عبارت در $\cos\alpha$ ضرب شده و $-1 \le \cos\alpha \le +1$ است و همواره از نظر عددی (بزرگی) کوچکتر می‌شود مگر (آنکه $\cos\alpha = 1$ باشد که آنگاه $S\alpha$ مساوی می‌شود) می‌شویم که $S\alpha$ عددی منفی بوده که باعث بزرگتر شدن آن شده است.

$\alpha \longrightarrow ?$ کدام ربع؟ (در نتیجه متوجه می‌شویم)



$$\frac{\cot\alpha}{\underbrace{S\alpha}_{\ominus}} > 0 \longrightarrow \cot\alpha\,\ominus \Longrightarrow \frac{\cos\alpha^{\,+}}{S\alpha_{\,-}} \Longrightarrow \alpha : \text{ربع } 4$$