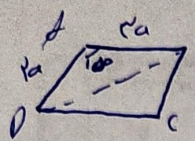
۲۰



$S_{ABCD} = 2S_{ABD} = 2 \times AD \times AB \times \sin A$ (۲)

$= 2 \times \frac{1}{2} \times 2a \times 2a \times \frac{1}{2} = 2a^2 = 8\sqrt{2} \Rightarrow a = 2\sqrt{2}$

محیط $= 2(AD + AB) = 2(5a) = 20\sqrt{2}$ ✓

$AD = BC$, $AB = DC$, $A = C = 150$ $\Rightarrow \hat{ABD} \cong \hat{BCD} \Rightarrow S_{ABD} = S_{BCD}$ $\Rightarrow S_{ABCD} = 2S_{ABD}$

$S_{ABCD} = S_{ABD} + S_{BCD}$

۲۱

$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin A = \frac{1}{2} \times 5 \times 7 \times \sin A = \frac{35}{2} \sin A$

$S_{ADE} = \frac{1}{2} AE \times AD \times \sin A = \frac{1}{2} \times 4 \times 7 \times \sin A = \frac{28}{2} \sin A$

$\Rightarrow |S_{ABE} - S_{ADE}|$ (۲)

$= \left|\frac{7}{2} \sin A\right| = 1.75$

$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30 \Rightarrow \tan A = \frac{\sqrt{3}}{3}$ ✓

$A = 30$

$A = 150$ یا $\sin A = -\frac{1}{2}$ غ ق ق

۲۲

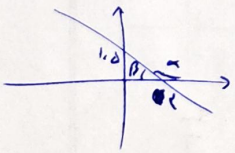
$\frac{1}{\sqrt{\cos^2 \alpha}} - \tan \alpha = \frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 - \sin \alpha}{|\cos \alpha|} = \frac{1}{|\cos \alpha|} - \frac{\sin}{|\cos \alpha|}$ (۲)

$\Rightarrow \frac{\sin \alpha}{|\cos \alpha|} = \frac{\sin \alpha}{-\cos \alpha} \Rightarrow |\cos \alpha| = -\cos \alpha \Rightarrow \cos < 0$

$\frac{|\sin \alpha|}{\cos \alpha} = -\tan \alpha = -\frac{\sin \alpha}{\cos \alpha} \Rightarrow |\sin \alpha| = -\sin \alpha \Rightarrow \sin \alpha < 0$

$\Rightarrow \alpha$ ✓ سوم

۲۳



$\tan \beta = \frac{4}{3}$

$\beta = 180 - \alpha$

$\tan(180 - \alpha) = \frac{4}{3} \Rightarrow \tan(\pi - \alpha) = \frac{4}{3}$ (۲)

$\Rightarrow \tan \alpha = -\frac{4}{3}$

$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha = \frac{1}{\tan \alpha} = \frac{1}{-\frac{4}{3}} = -\frac{3}{4}$ ✓

۱۵

$$\frac{2\cos(270-\theta)-2\sin(180-\theta)}{\sin(180+\theta)-\cos(270+\theta)} = \frac{2\cos(\frac{3\pi}{2}-\theta)-2\sin(\pi-\theta)}{\sin(\pi+\theta)-\cos(\frac{3\pi}{2}+\theta)} = \frac{-2\sin\theta-2\sin\theta}{-\sin\theta-\sin\theta}$$ (۲)

$$= -\frac{5\sin\theta}{-2\sin\theta} = \frac{5}{2}$$ ✓

۱۶

$$\frac{\sin(\frac{\pi}{2}+\alpha)\cdot-\sin(\alpha-\frac{\pi}{2})}{|\tan^2\alpha-1|} = \frac{\cos\alpha+\sin\alpha}{|\tan^2\alpha-1|} = \frac{\frac{2}{3}-\frac{\sqrt{5}}{3}}{|\frac{5}{4}-1|} = \frac{1-\sqrt{5}}{3}$$ ✓ (۲)

$\cos\alpha = \frac{2}{3} \Rightarrow \cos^2\alpha = \frac{4}{9} \Rightarrow 1-\cos^2\alpha = \sin^2\alpha = \frac{5}{9} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{3} \Rightarrow \sin\alpha = -\frac{\sqrt{5}}{3}$

$\Rightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{\sqrt{5}}{2}$

۱۷

$\sin\alpha = 2\cos\alpha \Rightarrow \tan\alpha = 2 \Rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 5 \Rightarrow \cos = \pm\frac{\sqrt{5}}{5}$ (۲)

$\Rightarrow \cos = -\frac{\sqrt{5}}{5}$ ✓

۱۸

$2mx+(m^2-1)y = 2 \Rightarrow y = -\frac{2m}{m^2-1}x + \frac{2}{m^2-1}$ (۲)

شیب $= \tan 60 = \sqrt{3} \Rightarrow \frac{-2m}{m^2-1} = \sqrt{3} \Rightarrow \sqrt{3}m^2+2m-\sqrt{3} = 0$

اختلاف مقادیر $m = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4+12}}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ ✓

۱۹

$\tan(\frac{\pi}{4}-x) = \frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x} = \frac{1-\tan x}{1+\tan x} = \frac{1-m}{m+2}$ (۲)

$\Rightarrow \frac{1-\tan x+1+\tan x}{1+\tan x} = \frac{1-m+2+m}{2+m} \Rightarrow \frac{1+\tan x}{2} = \frac{m+2}{3}$

$\Rightarrow 1+\tan x = \frac{4+2m}{3} \Rightarrow \tan x = \frac{2m+1}{3}$

$-\frac{\pi}{4} < x < \frac{\pi}{4} \Leftrightarrow -1 < \tan x < 1$

$-1 < \frac{2m+1}{3} < 1$

$\Rightarrow -3 < 2m+1 < 3$

$\Rightarrow -2 < m < 1$ ✓

$\tan(240^\circ)\cdot\cos(210^\circ) + \tan(450^\circ)\cdot\sin(135^\circ) = \tan(240^\circ)\cdot\cos(210^\circ) + \tan(135^\circ)\cdot\sin(90^\circ) \quad ; 1^\circ$

$= (-\sqrt{3})\cdot\left(-\frac{\sqrt{3}}{2}\right) + (-\sqrt{3})\left(\frac{\sqrt{2}}{2}\right) = \frac{3}{2} - \frac{\sqrt{6}}{2} > 0$ ✓

(٢)