نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$\log_n^m = a \qquad \log_{mn}^{m^2 n} = b \qquad [b] = ?$

$\log_{mn}^{m^2 n} = \log_{mn}^{mn} + \log_{mn}^{m} = 1 + \frac{\log_n^m}{\log_n^{mn}} = 1 + \frac{a}{\log_n^m + \log_n^n} = 1 + \frac{a}{1+a}$

$\rightarrow b = 1 + \overbrace{\frac{a}{1+a}}^{0<\,<1} \rightsquigarrow b = 1,\ldots \rightarrow [b] = 1$

۲

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}}^x}}$

① $x > 0$

② $\frac{x}{\log_{\frac{1}{2}}^x} > 0 \xrightarrow{x>0} \log_{\frac{1}{2}}^x > 0 \rightarrow x < 1$

①② $\rightarrow 0 < x < 1$

ب) $y = \frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$

	-1		2	
	$+$	$-$		$+$

$x^2 - 1 > 0 \rightarrow x > 1 \ \text{یا} \ x < -1$

$\rightarrow x \in (-\infty, -1) \cup (2, +\infty)$

۳

$2\log_x^a + \log_a^{\sqrt{a}} = 2 \rightarrow 2\log_x^a + \frac{1}{2}\log_a^x = 2 \rightarrow \frac{2}{\log_a^x} + \frac{\log_a^x}{2} = 2$

$t + \frac{1}{t} = 2 \rightarrow t^2 - 2t + 1 = 0 \rightarrow t = 1 \rightarrow \frac{2}{\log_x^a} = 1 \rightarrow \log_a^x = 2$

$x = 9 \rightarrow \log_a^9 = 2 \rightarrow a = 3$

۴

$\log 2 \simeq 0.3 \qquad \log 3 \simeq 0.4$

$\log 5 \simeq 0.7$

$\left[\log \frac{a}{2}\right] x^2 + \left[\log 9\right] x - \log 15$

$\log \frac{a}{2} = 0.7 - 0.4 = 0.3 \qquad \log 9 = 0.8 \qquad \log 15 = 0.4 + 0.7 = 1.1$

$\rightsquigarrow 0.3x^2 + 0.8x - 1.1 = 0 \rightsquigarrow 3x^2 + 8x - 11 = 0 \rightsquigarrow x^2 + 8x - 33 = 0$

$\rightarrow (x+11)(x-3) \rightarrow x_1 = \frac{3}{3}, -\frac{11}{3} \rightarrow \text{اختلاف} = \frac{14}{3}$

۵

$\log_2^7 = 2.8, \quad \log_5^2 = 10$

$\log_2^5 = 2$

$\log_5^7 \simeq \frac{\log_2^7}{\log_2^5} = \frac{2.8}{2} = 1.4$

$\log_{14}^{10} \simeq \frac{\log 5}{\log 14} = \frac{1.5}{1.9} = \frac{15}{19}$

$\log_{14}^5 = \log^7 5 * \log_5^2 = 5 + 1.4 = 1.9$

$\log 10 = \log 5 + \log 2 = 1.5$

$\log_2^3 = 1,4$ $\quad \log_3^5 = 1,5$ $\quad \log_2^3 \times \log_3^5 = \log_2^5 = 1,5 \times 1,4$

$$\log_{15}^{4} = \frac{\log_2^4}{\log_2^{15}} = \frac{\log_2^2 + \log_2^3}{\log_2^3 + \log_2^5} = \frac{1 + 1,4}{1,4 + (1,4 \times 1,5)} = \frac{2,4}{1,4 \times 2,5} = \frac{240}{14 \times 25} = \frac{12}{70} = \frac{6}{35}$$

۶

$\log_8^{11} = m \rightarrow m = \log_8^9 + \log_8^2 \rightarrow m = \log_8^9 + \frac{1}{3} \rightarrow m - \frac{1}{3} = \frac{2}{3}\log_2^3$

$\log_4^3 = \frac{2m-1}{2} \rightarrow \log_4^3 = \frac{2m-1}{4}$

$\log_4^{12} = \log_4^4 + \log_4^3 = \frac{2m-1}{4} + 1 = \frac{2m+3}{4}$

۷

$\left(\frac{1}{4}\right)^{2m-1} = \left(\frac{125}{8}\right)^{x^2} \rightarrow \left(\frac{2}{5}\right)^{2m-1} = \left(\frac{5}{2}\right)^{3x^2} \rightarrow -2m+1 = 3x^2$

$\rightarrow 3x^2 + 2x - 1 = 0 \rightarrow x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1) = 0 \rightarrow x = \frac{1}{3}, -1$

$x = -1$ غ ق ق $\rightarrow x = \frac{1}{3} \rightarrow \log_8^{(9x+1)} = \log_8^4 = \frac{2}{3}$

۸

$\log_2^3 = a \qquad \log_8^b = \frac{2}{3}(1+a)$

$\rightarrow \log_8^b = \frac{2}{3}\left(1 + \log_2^3\right) = \log_2^b = 2\left(\log_2^4\right) \rightarrow \log_2^b = \log_2^{24}$

$\rightarrow b = 24 \rightarrow \log_{10}^{2b-4} = \log_{10}^{44} = 2$

۹

$-4ax^2 + bx + \frac{1}{2}c = 0$ ~~کسر~~ $\rightarrow \frac{4a}{5} = \log 4$ $\begin{cases} 2a = b + c \\ \hookrightarrow b = 2a - c \end{cases}$

$\rightarrow \frac{4a}{b} = \log 4 \rightarrow \frac{4a}{2a-c} = \log 4 \rightarrow \frac{2a-c}{4a} = \log_4^{10} \rightarrow \frac{1}{2} - \frac{c}{4a} = \log_4^{10}$

$\rightarrow \frac{c}{4a} = \frac{1}{2} - \log_4^{10} \rightarrow \frac{c}{4a} = \log_4^2 - \log_4^{10} = \log_4^{\frac{1}{5}} \rightarrow \frac{c}{a} = 4\left(\log_4^{\frac{1}{5}}\right)$

۱۰

$\left(\frac{1}{\sqrt[3]{2}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{3}\left(\frac{c}{a}\right)} \rightarrow 2^{-\frac{1}{3}\left(\log_4^{\frac{1}{5}}\right)} = \left(2^{\log_4^{\frac{1}{5}}}\right)^{-\frac{1}{3}} = \left(\frac{1}{5}\right)^{-\frac{1}{3}} = \sqrt{5}$