[illegible] السیر A | ۱-

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$\log_n m = a \Rightarrow m = n^a$ $\qquad \log_{mn} m = \log_{n^a\cdot n} n^a = \log_{n^{a+1}} n^a$

$\log_{mn} m^2 n = b \Rightarrow \underbrace{\cancel{\log_{mn} mn}}_{1} + \log_{mn} m = b \qquad = \frac{a}{a+1}\cancel{\log_n n} \Rightarrow b = 1 + \frac{a}{a+1} \qquad 0<\frac{a}{a+1}<1$ ۲

$\boxed{[b] = 1}$

۲- ۲

الف) $\sqrt{\dfrac{x}{\log_{\frac12} x}}$ $\qquad x>0$ I

$\log_{\frac12} x \neq 0 \Rightarrow x \neq 1$ II

$\dfrac{x}{\log_{\frac12} x} \geq 0$ $\qquad$ $\begin{array}{ccccc} & 0 & & 1 & \\ - & | & + & | & - \end{array}$ III

$I\cap II\cap III = \boxed{(0,1)}$ $\quad x>0,\ x<1$

ب) $\dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$ $\qquad x^2-x-2>0$, $(x-2)(x+1)>0$ $\qquad$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array}$ $\qquad (-\infty,-1)\cup(2,+\infty)$ I

$\sqrt{x^2-1}+1 \neq 0$ همیشه مثبت

$x^2-1\geq 0 \qquad x^2\geq 1 \qquad (-\infty,-1]\cup[1,+\infty)$ II

$I\cap II = \boxed{(-\infty,-1)\cup(2,+\infty)}$

۳- ۲

$$\left.\begin{array}{l} 2\log_9 a + \log_a 3 = 2 \\ 2\log_{3^2} a + \log_a 3 = 2 \end{array}\right\} \log_3 a + \log_a 3 = 2, \quad t + \frac1t = 2 \xrightarrow{\times t} t^2+1-2t=0, \quad (t-1)^2=0, \quad t=1 \Rightarrow \log_a 3 = 1 \Rightarrow \boxed{a=3}$$

۴- ۲

$(\log\frac53)x^2 + (\log 9)x - \log 15 = 0 \qquad \log 9 = \log 3^2 = 2\log 3 = 2\times 0.4 = 0.8$

$\log\frac53 = \log 5 - \log 3 = \underbrace{\cancel{\log 10}}_{1} - \underbrace{\cancel{\log 2}}_{0.3} - \underbrace{\cancel{\log 3}}_{0.4} = 0.3 \qquad \underbrace{-\log 15 = -(\log 5 + \log 3)}_{= -1.1} = -(\underbrace{\cancel{\log 10}}_{1} - \underbrace{\cancel{\log 2}}_{0.3} + \underbrace{\cancel{\log 3}}_{0.4})$

$0.3x^2 + 0.8x - 1.1 = 0$

$3x^2 + 8x - 11 = 0$

$x^2 + 8x - 33 = 0$

$(x+11)(x-3) = 0 \qquad \frac{-11}{3} \quad \frac{3}{3} \Rightarrow \frac33 - (\frac{-11}{3}) = \boxed{\frac{14}{3}}$

۵- ۲

$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\cancel{\log_2 2} + \log_2 5}{\underbrace{\cancel{\log_2 2}}_{1} + \log_2 7} = \frac{1 + \frac{1}{\log_5 2}}{1 + \log_2 7} = \frac{1+\frac{1}{0.5}}{1+2.8} = \boxed{\frac{3}{3.8} \simeq 0.78}$$

۶- ۲

$$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \cancel{\log_3 3}}{\log_3 5 + \cancel{\log_3 3}} = \frac{\frac{1}{\log_2 3} + 1}{\log_3 5 + 1} = \frac{\frac{1}{1.6}+1}{1.5+1} = \frac{0.625+1}{1.5+1} = \boxed{\frac{1.625}{2.5} = 0.65}$$

۷- ۲

$\log_8 18 = m \Rightarrow \frac13\log_2 18 = m \Rightarrow \log_2 18 = 3m \qquad \underbrace{\cancel{\log_2 2}}_{1} + \log_2 3 + \log_2 3 = 3m$

$2\log_2 3 = 3m - 1$

$\log_2 3 = \frac{3m-1}{2}$

$\log_4 12 = ? \Rightarrow \frac12 \log_2 12 = ?$

$\frac12(\log_2 3 + \underbrace{\cancel{\log_2 4}}_{2}) = \frac12(\frac{3m-1}{2} + 2) = \boxed{\frac{3m+3}{4}}$

-۸ ۲

$$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2}$$

$$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2}$$

$$\left(\frac{2}{5}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2}$$

$$1-2x = 3x^2$$

$$3x^2+2x-1=0$$

$$x^2+2x-3=0$$

$$(x+3)(x-1)$$

$$\frac{-3}{3} \qquad \frac{1}{3}$$

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$$\log_8^{9x+1} = \log_8^{9\left(\frac{1}{3}\right)+1} = \log_8^{4} = \log_{2^3}^{2^2} = \frac{2}{3}\log_2^2 = \boxed{\frac{2}{3}}$$

-۹ ۲

$$\log_2^3 = a \Rightarrow 3 = 2^a$$

$$\log_8^b = \frac{2}{3}(1+a) \Rightarrow b = 8^{\frac{2}{3}(1+a)} \Rightarrow b = 2^{\cancel{3}\left(\frac{2}{\cancel{3}}\right)(1+a)} \Rightarrow b = 2^{2+2a}$$

$$b = 2^2 \times 2^a \times 2^a \Rightarrow b = 4\times3\times3 = \underline{\underline{36}} \qquad \log_{10}^{3b-8} = \log_{10}^{3\times36-8} = \log_{10}^{100} = 2$$

-۱۰ ۲

$$\frac{-b}{-4a} = \frac{1}{\log 4} \Rightarrow \frac{4a}{b} = \log 4$$

$$a = \frac{b+c}{2} \Rightarrow c = 2a-b$$

$$c = 2a - \frac{4a}{\log 4} \Rightarrow \left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}$$

$$\frac{c}{a} = 2 - \frac{4}{\log 4} \qquad \boxed{2^{\frac{-1}{8}\times\frac{c}{a}}}$$

$$2^{\frac{-1}{8}\times\left(2-\frac{4}{\log 4}\right)} = 2^{\frac{-1}{4}+\frac{1}{2\log 4}} \Rightarrow 2^{\log_{16}^{10}-\frac{1}{4}} = 2^{\log_{16}^{10}+\log_{16}^{\frac{1}{2}}} = 2^{\log_{16}^{5}}$$

$$\frac{1}{2\log 4} = \frac{1}{\log 16} = \log_{16}^{10}$$

$$\Rightarrow 5^{\log_{16}^{2}} = \boxed{5^{\frac{1}{4}} = \sqrt[4]{5}}$$