نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۰۳ کلاس .....

۱)

$$\log_{mn}^{m^2n} \xrightarrow{\text{تغییر پایه}} \frac{\log_n^{m^2n}}{\log_n^{mn}} = \frac{\log_n^{m^2}+\log_n^n}{\log_n^m+\log_n^n} = \frac{2\log_n^m+1}{\log_n^n+1} = \frac{2a+1}{a+1} = b$$

$$[b] = \left[\frac{2a+1}{a+1}\right] = \left[\frac{a+1}{a+1}+\frac{a}{a+1}\right] = \left[1+\frac{a}{a+1}\right] = 1$$

۲)

الف) $\frac{x}{\log_{\frac{1}{2}}^x} > 0 \rightarrow \frac{x}{-\log_2^x} > 0$

$x>0$

$\rightarrow -\log_2^x > 0 \rightarrow \log_2^x < 0$

$x<1$

$D: (0,1)$

ب) $x^2-x-2>0 \quad (x-2)(x+1)>0$

$\rightarrow x>2$ یا $x<-1$

$x^2-1\geq 0 \rightarrow x\geq 1$ یا $x\leq -1$

$\sqrt{x^2-1} \neq -1$ ✓

$D_f: (-\infty,-1)\cup(2,+\infty)$

۳)

$$2\log_x^a + \log_a^{\sqrt{x}} = 2 \rightarrow \log_{\sqrt{x}}^a + \log_a^{\sqrt{x}} \xrightarrow{\sqrt{x}=3} \frac{1}{\log_a^3} + \log_a^3 \xrightarrow{\log_a^3 = t}$$

$$\frac{1}{t} + t = 2 \rightarrow t^2-2t+1=0 \rightarrow (t-1)^2=0 \rightarrow t=1$$

$$\log_a^3 = 1 \rightarrow \boxed{a=3}$$

۴)

$$\log\frac{5}{4} = \log\frac{10}{2^4} - \log 3 = \log 10 - \log 2^2 - \log 3 = 1 - 0,3 - 0,4 = 0,3$$

$$\log 9 = \log 3^2 + \log 3^2 = 0,1 \quad , \quad \log 15 = \log 3 + \log 5 = \log 10 - \log 2 + \log 3 = 1,1$$

$$0,2x^2 + 0,1x - 1,1 = 0 \xrightarrow{\times 10} 2x^2+x-11=0 \rightarrow x^2+1x-22=0 \rightarrow (x+11)(x-2)=0$$

$x = -\frac{11}{2}$ | $x = \frac{22}{11} = 2$ (ق)

اختلاف $= 1-\left(-\frac{11}{2}\right) = \frac{13}{2}$ یا $-\frac{11}{2}-1 = -\frac{13}{2}$

۵)

$$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5+\log_2^2}{\log_2^7+\log_2^2} = \frac{2+1}{2,8+1} = \frac{3}{3,8} = \frac{30}{38} = \frac{15}{19}$$

$$\log_2^5 = \frac{1}{\log_5^2} = \frac{1}{\frac{1}{2}} = 2$$

$\log_{10}^{5} \times \log_{2}^{4} = \log_{2}^{5} = 1,4 \times 1,5 = 2,1$

$\log_{10}^{4} = \log_{10}^{2} + \log_{15}^{3} = \frac{1}{\log_{2}^{5\times 2}} + \frac{1}{\log_{3}^{5\times 3}} = \frac{1}{\log_{2}^{5}+\log_{2}^{2}} + \frac{1}{\log_{3}^{5}+\log_{3}^{3}} =$

$\frac{1}{2,1+1,4} + \frac{1}{1+1,5} = \frac{1}{4} + \frac{1}{2,5} = \frac{2,5+4}{10} = \frac{6,5}{10} = 0,65$

۶

$\log_{8}^{12} = m \to \frac{1}{3}\log_{2}^{12} = m \to \log_{2}^{9} + \log_{2}^{4} = 3m \to 2\log_{2}^{3} + 1 = 3m$

$\log_{2}^{3} = \frac{3m-1}{2}$ $\log_{4}^{12} = \frac{1}{2}\log_{2}^{12} = \frac{1}{2}(\log_{2}^{3} + \log_{2}^{4}) = 1 + \frac{3m-1}{4} = \frac{3m+3}{4}$

$\frac{3}{4}(m+1)$

۷

$\left(\frac{2}{5}\right)^{2m-1} = \left(\frac{125}{8}\right)^{m^2} \to \left(\frac{2}{5}\right)^{2m-1} = \left(\frac{2}{5}\right)^{-3m^2} \longrightarrow 2m-1 = -3m^2 \to 3m^2+2m-1 \longrightarrow$

$m=-1$ $m=\frac{1}{3}$ $\log_{8}^{4} = \frac{2}{3}$

۸

$2^a = 3^b$ $\left(2^2\right)^{\frac{2}{3}(1+a)} = 2^{4-4a} = b \to 2^2 \times (2^a)^2 = b \to b = 4\times 9 = 36$

$\log_{10}^{(3\times 27-1)} = \log_{11}^{10^2} = 2$

۹

۱۰