تکلیف   شهاب محقق کلاس دهم ریاضی

سوال ۱)

$\log_{n} m = a \qquad \log_{mn} m^{2}n = b \qquad \log_{mn} (mn\times n) = b \Rightarrow 1+\log_{mn} n = b$

$1+\frac{1}{\log_{n} mn} = b \Rightarrow 1+\frac{1}{1+\log_{n} m} = b \Longrightarrow 1+\frac{1}{1+a} = b \Rightarrow \frac{2a+1}{a+1} = b$

$\underbrace{\frac{a+1}{a+1}}_{1} < b < \underbrace{\frac{2a+2}{a+1}}_{2} \Rightarrow 1<b<2 \Rightarrow [b]=1$

سوال ۲)

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \qquad \log_{\frac{1}{2}} x = 0 \Rightarrow x=(\frac{1}{2})^{0}=1$

x		0		1	
y	$-$		$+$		$-$

I

چون $\log_{\frac{1}{2}} x$ نزولی هست پس در راست ریشه منفی می‌شود.

$\log_{\frac{1}{2}} x \to x>0 \quad II$

$\xrightarrow{I\cap II} 0<x<1 \Rightarrow D_f=(0,1)$

ب) $y=\frac{\log_{4}(x^{2}-x-2)}{\sqrt{x^{2}-1}+1} \to$ مخرج همواره مثبت هیچ گاه صفر نمی‌شود

$\log_{4}(x^{2}-x-2) \to x^{2}-x-2>0$

$(x-2)(x+1)>0$

x		-1		2	
y	$+$		$-$		$+$

I

$(-\infty,-1)\cup(2,+\infty)$

$\sqrt{x^{2}-1} \to x^{2}\geq 1 \to \begin{cases} x\geq 1 \\ x\leq -1 \end{cases} \quad II$

$\xrightarrow{I \text{ و } II}$ $D_f=(-\infty,-1)\cup(2,+\infty)$

سوال ۳)

$2\log_{x} a+\log_{a}\sqrt{x}=2 \qquad \log_{x} a^{2}+\frac{1}{2}\log_{a} x=2 \Rightarrow \begin{cases} a>0 \\ a\neq 1 \end{cases} \quad I$

$\log_{x} a^{2}+\log_{a^{2}} x=2 \Rightarrow \log_{x} a^{2}+\frac{1}{\log_{x} a^{2}}=2 \xrightarrow{\log_{x} a^{2}=t} t+\frac{1}{t}=2$

$t^{2}-2t+1=0 \to (t-1)^{2}=0 \to t=1 \qquad \log_{x} a^{2}=1 \xrightarrow{x=9} \log_{9} a^{2}=1$

$a^{2}=9 \xRightarrow{I} \begin{cases} a=3 & \text{ق ق} \checkmark \\ a=-3 & \text{غ ق ق} \end{cases}$

سوال ۴)

$\log 2 \simeq 0/3$ $\quad \log 3 \simeq 0/4$ $\qquad (\log \frac{5}{3})x^2 + (\log 9)x - \underbrace{\log 15}_{\log 3\times 5} = 0$

$1 - \log 2 = \log 5 \Rightarrow \log 5 = 1 - 0/3 = 0/7$

$(\log 5 - \log 3)x^2 + (2\log 3)x - (\log 3 + \log 5) = 0$

$(0/7 - 0/4)x^2 + (2\times 0/4)x - (0/4 + 0/7) = 0 \qquad 0/3x^2 + 0/8x - 1/1 = 0$

$3x^2 + 8x - 11 = 0 \Rightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{64+132}}{3} = \frac{14}{3}$

سوال ۵)

$\log_2 5 = 2$

$\log_2 7 = 2/8 \qquad \log_5 2 = 0/5 \qquad \log_{14} 10 = ? \qquad \log_5 7 = \frac{\log_2 7}{\log_2 5} = \frac{2/8}{2} = \underline{1/4}$

$\log_{14} 5\times 2 = \log_{14} 5 + \log_{14} 2 = \frac{1}{\log_5 14} + \frac{1}{\log_2 14} = \frac{1}{\log_5 7 + \log_5 2} + \frac{1}{1 + \log_2 7}$

$= \frac{1}{1/4 + 0/5} + \frac{1}{1 + 2/8} = \frac{1}{1/9} + \frac{1}{3/8} = \frac{2+1}{3/8} = \frac{3}{3/8} = \frac{30}{38} = \frac{15}{19}$

سوال ۶)

$\log_3 2 = \frac{1}{1/6}$

$\log_3 5 = 1/5 \qquad \log_2 3 = 1/6 \qquad \log_{15} 6 = ? \qquad \log_2 5 = \frac{\log_3 5}{\log_3 2} = \frac{1/5}{\frac{1}{1/6}} = \frac{12}{5} = 2/4$

$\log_{15} 3\times 2 = \log_{15} 3 + \log_{15} 2 = \frac{1}{1 + \log_3 5} + \frac{1}{\log_2 5 + \log_2 3} = \frac{1}{1 + 1/5} + \frac{1}{2/4 + 1/6}$

$\frac{1}{2/5} + \frac{1}{4} = \frac{4 + 2/5}{10} = \frac{6/5}{10} = 0/65$

سوال ۷)

$\log_8 18 = m \qquad \log_4 12 \qquad \frac{1}{3}\log_2 18 = \frac{1}{3}(\log_2 2\times 3^2) = \frac{1}{3}(1 + 2\log_2 3) = m$

$\Rightarrow \frac{3m-1}{2} = \log_2 3 \quad , \quad \log_4 12 = \frac{1}{2}\log_2 12 = \frac{1}{2}(2 + \log_2 3)$

$\frac{1}{2}\left(\frac{3m-1+4}{2}\right) = \frac{3m+3}{4} = \frac{3}{4}(m+1)$

سوال ۸

$$(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2}$$

$$\left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2+2x-1=0 \overset{\text{ریشه}}{\Rightarrow} (x+3)(x-1) \begin{cases} x=\frac{-3}{3}=-1 & \text{غ ق ق} \\ x=\frac{1}{3} & \text{ق ق} \end{cases}$$

$$\log_8^{9x+1} \Rightarrow 9x+1>0 \quad x>-\frac{1}{9} \qquad \log_8^{\frac{9}{3}+1} = \log_8^{4} = \frac{2}{3}\log_2^{2} = \boxed{\frac{2}{3}}$$

سوال ۹

$$\log_2^{3}=a \qquad \log_8^{b}=\frac{2}{3}(1+a) \qquad \log^{3b-8} = ?$$

$$\log_8^{b} = \frac{2}{3}(1+\log_2^{3}) \Rightarrow \frac{1}{3}\log_2^{b} = \frac{2}{3}\log_2^{6} \Rightarrow \log_2^{b} = \log_2^{36}$$

$$b=36$$

$$\log_{10}^{3(36)-8} = \log_{10}^{100} = \boxed{2}$$

سوال ۱۰

$$-4ax^2+bx+\frac{1}{2}c=0 \qquad \frac{1}{\alpha+\beta} = \log^{4} \qquad a=\frac{b+c}{2} \ \ \mathrm{I}$$

$$\frac{1}{\frac{-b}{2(-4a)}} = \log_{10}^{4} \Rightarrow \frac{8a}{b} = \log_{10}^{4} \Rightarrow \frac{4a}{b} = \log^{2} \Rightarrow 2a = \frac{\log^{2}}{2}b$$

$$\Rightarrow a = \frac{\log^{2}}{4}b$$

$$\overset{\mathrm{I}}{\Rightarrow} 2a=b+c \Rightarrow b\left(\frac{\log^{2}}{2}-1\right)=c$$

$$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{b\left(\frac{\log 2}{2}-1\right)}{b\left(\frac{\log 2}{4}\right)}} = \left(2^{-\frac{1}{8}}\right)^{\frac{2\log 2-4}{\log 2}} = 2^{-\frac{2}{8}\left(\frac{\log 2-2}{\log 2}\right)}$$

$$= 2^{-\frac{1}{4}\log_2^{\frac{2}{100}}} = 2^{\log_2^{\left(\frac{2}{100}\right)^{-\frac{1}{4}}}} = \left(\frac{2}{100}\right)^{-\frac{1}{4}\log_2^{2}} = \left(\frac{100}{2}\right)^{\frac{1}{4}}$$