

$$\log_n^m = a \rightarrow m = n^a$$

$$\log_{mn}^{m^r n} = b \xrightarrow{m=n^a} \log_{n^{a+1}}^{n^{ra+1}} = b \rightarrow n^{(a+1) \times b} = n^{ra+1}$$

$$\rightarrow b = \frac{ra+1}{a+1} = \frac{ra+r-1}{a+1} = r - \frac{1}{a+1} \xrightarrow{a>0} \frac{1}{a+1} < 1 \rightarrow 1 < r - \frac{1}{a+1} < r \rightarrow [b] = 1 \quad \checkmark$$

الف) $y = \sqrt{\frac{n}{\log \frac{1}{r}}}$ $\rightarrow \frac{n}{\log \frac{1}{r}} \geq 0 \rightarrow \frac{n}{\log \frac{1}{r}} \geq 0 \rightarrow \frac{n}{\log \frac{1}{r}} \geq 0 \rightarrow \frac{n}{\log \frac{1}{r}} \geq 0 \rightarrow D_f = (0, 1) \quad \checkmark$

ب) $y = \frac{\log n^r - n + r}{\sqrt{n^2 - 1} + 1} \rightarrow \begin{cases} n^r - n + r > 0 & \frac{-1 \pm r}{\pm 1 - \pm 1} \\ n^2 - 1 \geq 0 \rightarrow n^2 \geq 1 \rightarrow n \geq 1 \pm n \leq -1 \end{cases} \rightarrow D_f = (-\infty, -1) \cup (1, +\infty) \quad \checkmark$

$$r \log_n^a + \log_a^{\sqrt{n}} = r \rightarrow \log_n^{ar} + \frac{r}{r} \log_a^{\sqrt{n}} = r \rightarrow \log_n^{ar} + \log_{ar}^n = r$$

$$\log_n^{ar} = t \quad t + \frac{1}{t} = r \rightarrow t^r - rt + 1 = 0 \rightarrow t = 1 \rightarrow \log_n^{ar} = 1$$

$$\xrightarrow{n=9} \log_9^{ar} = 1 \rightarrow a^r = 9 \rightarrow \begin{cases} a = +3 \quad \checkmark \\ a = -3 \quad \times \end{cases} \quad (a > 0)$$

$$\left(\log \frac{3}{2}\right)n^r + (\log 4)n - \log 18 = 0 \quad \log 3 = 0.18, \log 2 = 0.18, \log 8 = 1 - \log 2 = 0.18$$

$$\left(\log 8 - \log 3\right)n^r + (r \log 3)n - (\log 3 + \log 8) = 0 \rightarrow 0.18n^r + 0.18n - 1.18 = 0$$

$$a+b+c=0 \rightarrow n=1 \pm n = \frac{-1.1}{0.18} = -\frac{11}{2} \rightarrow \left|1 - \left(-\frac{11}{2}\right)\right| = \frac{13}{2} \quad \checkmark$$

$$\log_2^V = r.18 \rightarrow \log V = r.18 \log 2, \log_8^r = 0.18 \rightarrow \log 8 = r \log 2$$

$$\rightarrow \log_{12}^{1.2} = \frac{\log 1.2}{\log 12} = \frac{\log 8 + \log 2}{\log V + \log 2} = \frac{r \log 2}{r.18 \log 2} = \frac{r}{r.18} \quad \checkmark \quad \log_{12}^{1.2} = \frac{\log \frac{1.2}{12}}{\log \frac{12}{2}}$$

$$= \frac{\log 2 + \log 2}{\log V + \log 2} = \frac{r+1}{r.18+1} = \frac{r}{r.18} \quad \checkmark$$

$$\log_{\frac{\delta}{r}} \delta = 1, \delta \rightarrow \log \delta = 1, \delta \log r \quad \} \quad \log r = 1, \gamma \rightarrow \log r = \frac{\gamma}{\lambda} \log r$$

$$\log_{\frac{\gamma}{\delta}} \gamma = \frac{\log \gamma}{\log \frac{\gamma}{\delta}} = \frac{\log r + \log r}{\log r + \log \delta} = \frac{\frac{\gamma}{\lambda} \log r}{1, \delta \log r} = \frac{\gamma}{\gamma_0} \checkmark$$

(r)

$$\log_{\frac{1}{\lambda}} \lambda = m = \log_{\frac{1}{\lambda}} \lambda + \log_{\frac{1}{\lambda}} r = \log_{\frac{1}{\lambda}} \lambda + \frac{1}{r} = m \rightarrow \log_{\frac{1}{\lambda}} \lambda = m - \frac{1}{r}$$

$$\rightarrow \frac{r}{r} \log r = m - \frac{1}{r} \rightarrow \log r = \frac{r}{r} m - \frac{1}{r} \rightarrow \log \frac{1}{r} = \log \frac{r}{\varepsilon} + \log \frac{\varepsilon}{r}$$

$$= \frac{1}{r} \log r + 1 = \frac{r}{\varepsilon} m - \frac{1}{\varepsilon} + 1 = \frac{r}{\varepsilon} m + \frac{r}{\varepsilon} = \frac{r m + r}{\varepsilon} \checkmark$$

(r)

$$\left(\frac{r}{\delta}\right)^{r n - 1} = \left(\frac{1, \delta}{\lambda}\right)^{n r} \rightarrow \frac{r^{r n - 1}}{\delta^{r n - 1}} = \frac{\delta^{r n r}}{r^{r n r}} \rightarrow r^{r n r + r n - 1} = \delta^{r n r + r n - 1}$$

$$\rightarrow r n r + r n - 1 = 0 \rightarrow n = -1, n = \frac{1}{r} \xrightarrow{r n + 1 > 0} n = \frac{1}{r} \rightarrow \log_{\frac{1}{\lambda}} \lambda^{r n + 1} = \log_{\frac{1}{\lambda}} \lambda$$

$$= \frac{r}{r} \log r = \frac{r}{r} \checkmark$$

(r)

$$\log r = a \quad \log_{\frac{b}{r}} b = r(1+a) \rightarrow \frac{1}{r} \log_{\frac{b}{r}} b = 1+a$$

$$\rightarrow \frac{1}{r} \log_{\frac{b}{r}} b = \log_{\frac{b}{r}} r + \log_{\frac{b}{r}} r \rightarrow \frac{1}{r} \log_{\frac{b}{r}} b = \log_{\frac{b}{r}} r \rightarrow \log_{\frac{b}{r}} b = r \log_{\frac{b}{r}} r$$

$$\rightarrow \log_{\frac{b}{r}} b = \log_{\frac{b}{r}} r^r \rightarrow b = r^r \rightarrow \log(r^r b - 1) = \log(100) = (r) \checkmark$$

(r)

$$- \varepsilon a n^r + b n + \frac{1}{r} c = 0 \rightarrow \frac{1}{\alpha + \beta} = \frac{\varepsilon a}{b} = \log \varepsilon, \quad \begin{matrix} r a = c + b \\ b = r a - c \end{matrix}$$

$$\rightarrow \frac{\varepsilon a}{r a - c} = \log r \rightarrow \frac{r a - c}{\varepsilon a} = \log \frac{1}{\varepsilon} \rightarrow \frac{1}{r} - \frac{c}{\varepsilon a} = \log \frac{1}{\varepsilon} \rightarrow \frac{c}{\varepsilon a} = \frac{1}{r} - \log \frac{1}{\varepsilon}$$

$$= \log r - \log \frac{1}{\varepsilon} = \log \frac{\varepsilon}{r} \rightarrow \left(\frac{c}{a} = \varepsilon \log \frac{\varepsilon}{r}\right) \rightarrow \left(\frac{1}{\sqrt{r}}\right)^{\frac{c}{a}} = \left(r^{-\frac{1}{r}}\right)^{\frac{c}{a}} = \left(r^{\log \frac{\varepsilon}{r}}\right)^{-\frac{1}{r}}$$

$$= \left(\frac{1}{r}\right)^{-\frac{1}{r}} = \left(\frac{1}{r}\right)^{-\frac{1}{\varepsilon}} = \delta^{\frac{1}{\varepsilon}} = \sqrt{\delta} \checkmark$$

(r)