

$$\log_n^m = a \rightarrow m = n^a$$

$$\log_{mn}^{m'n} = \log_{n^{a+1}}^{n^{a+1}} = \frac{a+1}{a+1} \log_n^n = \frac{a+1}{a+1} = b$$

$$b = \frac{a+1}{a+1} \rightarrow [b] = \left[\frac{a}{a+1} + 1 \right] = 1 + \left[\frac{a}{a+1} \right] \xrightarrow{a > 0} [b] = 1$$

$$\frac{x}{\log \frac{x}{\frac{1}{x}}} \geq 0, \log \frac{x}{\frac{1}{x}} \neq 0$$

$$x > 0 \Rightarrow \log \frac{x}{\frac{1}{x}} > 0 \rightarrow x > \frac{1}{x} \rightarrow x > 1 \quad (3)$$

$$\Rightarrow \frac{1}{x} \neq x \rightarrow x \neq 1 \quad (1)$$

$$x > 0 \quad (2)$$

$$(1) \cap (2) \cap (3) \Rightarrow D = [1, +\infty)$$

$$x^2 - x - 2 > 0 \rightarrow (x+1)(x-2) > 0 \rightarrow \frac{-1}{2} < x < \frac{2}{2} \rightarrow -\infty < x < -1 \cup 2 < x < +\infty \quad (4)$$

$$\sqrt{x^2 - 1} + 1 \neq 0 \quad \text{همیشه بزرگتر از 0}, \sqrt{x^2 - 1} \geq 0 \rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow -1 \leq x \leq 1 \quad (5)$$

$$(1) \cap (5) \rightarrow D = \emptyset$$

$$2 \log_x^a + \log_a^{\sqrt{x}} = 2, x=9 \Rightarrow 2 \log_9^a + \log_a^3 = 2 \rightarrow \frac{2}{3} \log_9^a + \log_a^3 = 2$$

$$\log_9^a = 2 - \log_a^3 \rightarrow \log_9^a + \log_a^3 = 2 \Rightarrow \frac{\log_9^a}{\log_9^3} + \frac{\log_9^3}{\log_9^a} = \frac{\log_9^a}{1} + \frac{1}{\log_9^a} = 2, \log_9^a = t$$

$$\Rightarrow t + \frac{1}{t} = 2 \rightarrow t^2 - 2t + 1 = 0 \xrightarrow{t=1} (t-1)^2 = 0 \Rightarrow \log_9^a = 1 \Rightarrow a = 9$$

$$= \left(\log^{\frac{1}{2}} - \log^{\frac{1}{4}} \right) x^{\frac{1}{2}} \left(\log^{\frac{3}{4}} \right) x - \left(\log^{\frac{3}{4}} + \log^{\frac{1}{4}} \right) = 0 \rightarrow (1 - \frac{1}{2} - \frac{1}{4}) x^{\frac{1}{2}} + \frac{1}{2} x (\frac{1}{4}) x - \frac{1}{4} - \log^{\frac{1}{4}}$$

$$\log^{\frac{1}{4}} = \log^{\frac{1}{2}} = \log^{\frac{1}{4}} - \log^{\frac{1}{4}} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\rightarrow \frac{3}{4} x^{\frac{1}{2}} + \frac{1}{8} x - \frac{1}{4} = 0$$

$$\text{اختلاف مربع} = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{0.75^2 + 4 \cdot \frac{1}{8} \cdot \frac{1}{4}}}{\frac{1}{8}} = \frac{1}{0.75} = \frac{4}{3}$$

$$\log_{16}^{10} = \frac{\log_4^{10}}{\log_4^{16}} = \frac{\log_4^2 + \log_4^8}{\log_4^2 + \log_4^8} = \frac{1+2}{1+2} = \frac{3}{3} = 1$$

$$\log_{16}^2 = \frac{1}{4} \rightarrow \log_4^2 = 2$$

$$\log_{10}^y = \frac{\log_v^y}{\log_v^{10}} = \frac{\log_v^y + \log_v^w}{\log_v^y + \log_v^w} = \frac{1+14}{14+10} = \frac{15}{24} = \boxed{\frac{5}{8}}$$

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$$\log_{v^r}^{10} = m \rightarrow \log_v^{10} = r_m = \log_v^9 + \log_v^r = r \log_v^r + 1 = r_m \Rightarrow \log_v^r = \frac{r_m - 1}{r}$$

$$\log_{v^r}^{10} = \frac{1}{r} \log_v^{10} = \frac{1}{r} (\log_v^9 + \log_v^r) = \frac{1}{r} (r + \log_v^r) = \frac{1}{r} (r + \frac{r_m - 1}{r}) = \boxed{\frac{r_m + r}{r^2}}$$

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$$(\cdot)^{r_{n-1}} = \frac{r^{r_{n-1}}}{\omega^{r_{n-1}}} = \left(\frac{\omega^r}{r^r} \right)^{r_{n-1}} = \frac{\omega^{r_{n-1}}}{r^{r_{n-1}}} \rightarrow r^{r_{n-1} + r_{n-1}} = \omega^{r_{n-1} + r_{n-1}} \rightarrow r_{n-1} + r_{n-1} = 0 \begin{cases} x = -1 \\ n = \frac{1}{r} \end{cases}$$

$$\log_{10}^{q_{n+1}} = \frac{1}{r} \log_v^{q_{n+1}} \begin{cases} x = \frac{1}{r} \rightarrow \frac{1}{r} \log_v^r = \frac{1}{r} \times r = \boxed{\frac{r}{r}} \\ x = -1 \rightarrow \frac{1}{r} \log_v^{-1} \notin \mathbb{R} \end{cases}$$

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$$\log_{v^r}^b = \frac{1}{r} \log_v^b = \frac{r}{r} (1+a) \Rightarrow \log_v^b = r(a+1) \rightarrow b = r^a \times r^{\frac{a=\log_v^w}{r^a=r}} b = r \times r = r^2$$

$$\Rightarrow \log_{10}^{r^{b-1}} = \log_{10}^{10^{10-1}} = \log_{10}^{100} = \boxed{2}$$

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