

$$\log_{mn} m^n = \frac{\log m^n}{\log mn} = \frac{n \log m + 1}{\log m + 1} \quad \log m = a \rightarrow \log_{mn} m^n = \frac{na + 1}{a + 1} = b$$

$$\rightarrow \frac{na + 1}{a + 1} = n - \frac{1}{a + 1} \xrightarrow{a=0} \left[n - \frac{1}{a + 1} \right] = n \left[-\frac{1}{a + 1} \right] \xrightarrow{-\frac{1}{a+1} \rightarrow b} [b] = 1$$

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الف) $\log \frac{x}{\sqrt{x}} \rightarrow x > 0 \left\{ \sqrt{\frac{x}{\log \frac{x}{\sqrt{x}}}} \rightarrow \frac{x}{\log \frac{x}{\sqrt{x}}} > 0 \right.$

$\log \frac{x}{\sqrt{x}} = 0 \rightarrow x = 1$

$\rightarrow x \in (0, 1)$

$\rightarrow \log \frac{x^2 - x - 2}{x} \rightarrow x^2 - x - 2 > 0$

$\rightarrow x \in (-\infty, -1) \cup (2, +\infty) \quad \textcircled{1}$

$\sqrt{x^2 - 1} \rightarrow x^2 - 1 > 0 \rightarrow x \in (-\infty, -1) \cup (1, +\infty) \quad \textcircled{2}$

$\rightarrow x \in (-\infty, -1) \cup (2, +\infty)$

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$n \log a = \log \frac{a}{\sqrt{a}} = \frac{1}{\log \sqrt{a}} \rightarrow n \log a + \log \sqrt{a} = n$

$\rightarrow \frac{1}{\log \sqrt{a}} + \log \sqrt{a} = n \quad \log \sqrt{a} = t \rightarrow \frac{1}{t} + t = n \rightarrow t = 1$

$\rightarrow \log \sqrt{a} = 1 \xrightarrow{x=9} a = 9$

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$\log \frac{10}{\sqrt{2}} = \log 10 - \log \sqrt{2} = 1 - 0.15 = 0.85 \rightarrow \log \frac{10}{\sqrt{2}} - \log 2 = \log \frac{5}{\sqrt{2}} = 0.75$

$\log 9 = 2 \log 3 = 0.55, \log 15 = \log \frac{30}{2} = \log 10 + \log 3 - \log 2 = 1.1$

$\rightarrow (\log \frac{5}{\sqrt{2}}) x^2 + (\log 9) x - \log 15 = 0 \rightarrow \frac{\sqrt{\Delta}}{a} = \frac{\sqrt{(0.55)^2 + 4(0.75)(1.1)}}{0.75} = \frac{1.7}{0.75} = \frac{14}{5}$

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$\log \frac{1}{5} = 0.3 \rightarrow \log \frac{1}{2} = 0.3, \log \frac{1}{3} = 0.18$

$\log \frac{1}{12} = \frac{\log \frac{1}{2}}{\log \frac{1}{3}} = \frac{\log \frac{1}{2} + \log \frac{1}{3}}{\log \frac{1}{2} + \log \frac{1}{3}} = \frac{0.3}{0.3 + 0.18} = \frac{30}{48} = \frac{5}{8} = 0.625$

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$$\log_{10} 4 = \frac{\log 4}{\log 10} = \frac{\log 2 + \log 2}{\log 2 + \log 5} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5}} = \frac{1}{2} = 0,5$$

$$\log_2 10 = 1,4 \rightarrow \log_{10} 2 = \frac{1}{1,4}$$

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$$\log_{10} 100 = \frac{1}{2} \log_{10} 100 = \frac{1}{2} (2 \log_{10} 10 + \log_{10} 10)$$

$$\log_{10} 100 = m \rightarrow 2m = \log_{10} 100 = \log_{10} 10 + 2 \log_{10} 10 \rightarrow \log_{10} 10 = \frac{2m-1}{2}$$

$$\rightarrow \frac{1}{2} (2 \log_{10} 10 + \log_{10} 10) = \frac{2m-1}{2}$$

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$$(0,1)^{2x-1} = \left(\frac{1}{10}\right)^{1-2x}, \left(\frac{1}{10}\right)^{2x} = \left(\frac{1}{10}\right)^{2x}$$

$$\rightarrow (0,1)^{2x-1} = \left(\frac{1}{10}\right)^{2x} \rightarrow 1-2x = 2x \rightarrow 2x+2x-1=0 \rightarrow x=-1, \frac{1}{10}$$

$$\rightarrow \log_{10}^{(9x+1)} = \log_{10} 10 = \frac{1}{10}$$

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$$\log_{10} b = \frac{1}{2} (1+x) = \frac{1}{2} (1 + \log_{10} 2) = \frac{1}{2} (\log_{10} 4) = \log_{10} 2 \rightarrow b = 2$$

$$\log_{10}^{2b-1} = \log_{10} 10 = 1$$

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$$\log_{10} 10 = \frac{a}{b} \rightarrow \log_{10} \sqrt{10} = \frac{a}{b} \text{ (} a=b+c \rightarrow c=a-b \rightarrow \frac{c}{a} = 1 - \frac{b}{a} \text{)}$$

$$\rightarrow \left(\frac{1}{\sqrt{10}}\right)^{\frac{c}{a}} = \sqrt{10}^{\frac{b}{a}-1} \xrightarrow{\log_{10} \frac{b}{a}} \sqrt{10}^{\log_{10} \frac{1}{\sqrt{10}}} = \sqrt{10} = 10^{\log_{10} \frac{1}{\sqrt{10}}} = \sqrt{10}$$

$$= \sqrt{10} \div \sqrt{10} = 1$$

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