نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس ..... A .....

۱ (۲)

$$\log_{mn}^{m^2n} = \frac{\log_n^{m^2n}}{\log_n^{mn}} = \frac{2\log_n^m + 1}{\log_n^m + 1} \xrightarrow{\log_n^m = a} \log_{mn}^{m^2n} = \frac{2a+1}{a+1} = b$$

$$\rightarrow \frac{2a+1}{a+1} = 2 - \frac{1}{a+1} \xrightarrow{a>0} \left[2 - \frac{1}{a+1}\right] = 2\left[-\frac{1}{a+1}\right] \xrightarrow{-1<-\frac{1}{a+1}<0} [b] = 1 \checkmark$$

۲ (۲)

الف) $\log_{\frac{1}{2}}^{x} \rightarrow x>0 \;\{\; \sqrt{\frac{x}{\log_{\frac{1}{2}}^{x}}} \rightarrow \frac{x}{\log_{\frac{1}{2}}^{x}} \geq 0$

$\log_{\frac{1}{2}}^{x} = 0 \rightarrow x = 1$

x	0	1
	∄	0
	+	−

$\rightarrow x \in (0,1) \checkmark$

ب) $\log_{2}^{x^2-x-2} \rightarrow x^2-x-2>0$

$\rightarrow x \in (-\infty,-1) \cup (2,+\infty)$ ①

$\sqrt{x^2-1} \rightarrow x^2-1 \geq 0 \rightarrow x \in (-\infty,-1) \cup (1,+\infty)$ ② $\{\sqrt{x^2-1}+1 \neq 0 \rightarrow x \in R$

$\rightarrow x \in (-\infty,-1) \cup (2,+\infty) \checkmark$

۳ (۲)

$$2\log_x^a = \log_{\sqrt{x}}^{a} = \frac{1}{\log_a^{\sqrt{x}}} \rightarrow 2\log_x^a + \log_a^{\sqrt{x}} = 2$$

$$\rightarrow \frac{1}{\log_a^{\sqrt{x}}} + \log_a^{\sqrt{x}} = 2 \xrightarrow{\log_a^{\sqrt{x}} = t} \frac{1}{t} + t = 2 \rightarrow t = 1$$

$$\rightarrow \log_a^{\sqrt{x}} = 1 \xrightarrow{x=9} a = 3 \checkmark$$

۴ (۲)

$$\log\frac{10}{3} = \log 10 - \log 3 = 1 - 0.4 = 0.6 \rightarrow \log\frac{10}{3} - \log 2 = \log\frac{5}{3} = 0.3$$

$$\log 9 = 2\log 3 = 0.8 \;,\; \log 15 = \log\frac{30}{2} = \log 10 + \log 3 - \log 2 = 1.1$$

$$\rightarrow (\log\tfrac{5}{3})x^2 + (\log 9)x - \log 15 = 0 \rightarrow \frac{\sqrt{\Delta}}{a} = \frac{\sqrt{(0.8)^2 + 4(1.1)(0.3)}}{0.3} = \frac{1.4}{0.3} = \frac{14}{3} \checkmark$$

۵ (۲)

$$\log_5^2 = 0.5 \rightarrow \log\frac{5}{2} = 2 \;,\; \log\frac{7}{2} = 2.8$$

$$\log_{14}^{10} = \frac{\log\frac{10}{2}}{\log 14} = \frac{\log\frac{5}{2} + \log\frac{7}{2}}{\log\frac{7}{2} + \log 2} = \frac{3}{3.8} = \frac{30}{38} = \frac{15}{19} \checkmark$$

۶ (۲)

$\log_{10}^{4} = \frac{\log_{3}^{4}}{\log_{3}^{10}} = \frac{\log_{3}^{2^2} + \log_{3}^{2}}{\log_{3}^{2} + \log_{3}^{5}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} = 0/5$ ✓

$\log_{2}^{3} = 1/4 \rightarrow \log_{3}^{2} = \frac{5}{8}$

۷ (۲)

$\log_{4}^{12} = \frac{1}{2}\log_{2}^{12} = \frac{1}{2}(2\log_{2}^{2} + \log_{2}^{3})$

$\log_{8}^{18} = m \rightarrow 3m = \log_{2}^{18} = \log_{2}^{2} + 2\log_{2}^{3} \rightarrow \log_{2}^{3} = \frac{3m-1}{2}$

$\rightarrow \frac{1}{2}(2\log_{2}^{2} + \log_{2}^{3}) = \boxed{\frac{3m+3}{4}}$ ✓

۸ (۲)

$(0/4)^{2x-1} = (\frac{5}{2})^{1-2x}$, $(\frac{125}{8})^{x^2} = (\frac{5}{2})^{3x^2}$ غ ق ق

$\rightarrow (0/4)^{2x-1} = (\frac{125}{8})^{x^2} \rightarrow 1-2x = 3x^2 \rightarrow 3x^2+2x-1=0 \rightarrow x = -1, \frac{1}{3}$

$\rightarrow \log_{8}^{(9x+1)} = \log_{8}^{4} = \frac{2}{3}$ ✓

۹ (۲)

$\log_{8}^{b} = \frac{2}{3}(1+a) = \frac{2}{3}(1+\log_{2}^{3}) = \frac{2}{3}(\log_{2}^{6}) = \log_{8}^{3^2 6} \rightarrow b = 3^2 6$ ✓

$\log_{15}^{3b-8} = \log_{15}^{100} = 2$ ✓

۱۰ (۲)

$\log_{10}^{2} = \frac{2a}{b} \rightarrow \log_{10}^{\sqrt{2}} = \frac{a}{b}$, $2a = b+c \rightarrow c = 2a-b \rightarrow \frac{c}{a} = 2 - \frac{b}{a}$

$\rightarrow (\frac{1}{\sqrt{2}})^{\frac{c}{a}} = \sqrt{2}^{\frac{b}{a}-2} \xrightarrow{\log_{\frac{1}{\sqrt{2}}}} \sqrt{2}^{\log_{\sqrt{2}}^{10}} \div \sqrt[4]{2} = 10^{\log_{\sqrt{2}}^{\sqrt[4]{2}}} \div \sqrt[4]{2}$

$= \sqrt[4]{10} \div \sqrt[4]{2} = \sqrt[4]{5}$ ✓