۱)

$$\left.\begin{array}{l}\log_n^m = a \Rightarrow m = n^a \\ \log_{mn}^{m^2 n} = b\end{array}\right\} \Rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \Rightarrow \frac{2a+1}{a+1}\log_n^n = b$$

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$$\left.\begin{array}{l} \Rightarrow b = \dfrac{2a+1}{a+1} \\ \overset{a>0}{\Rightarrow} \dfrac{a+1}{a+1} < \dfrac{2a+1}{a+1} < \dfrac{2a+2}{a+1} \Rightarrow 1 < \dfrac{2a+1}{a+1} < 2 \end{array}\right\} \Rightarrow 1 < b < 2 \Rightarrow \underline{[b] = 1}$$

۲)

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}}^{x}}} \rightarrow x > 0$ $\qquad \log_{\frac{1}{2}}^{x} \neq 0 \Rightarrow x \neq 1$

x	$-\infty$	0	1	$+\infty$
x	$-$	0 $+$	$+$	
$\log_{\frac{1}{2}}^{x}$	غ ت	$+$	0 $-$	
$\dfrac{x}{\log_{\frac{1}{2}}^{x}}$	غ ت	$+$	0 $-$	

$\Leftarrow$ $\underline{D_f = (0,1]}$

ب) $y = \dfrac{\log_2^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$

$$\begin{cases} x^2-x-2>0 \Rightarrow (x+1)(x-2)>0 \Rightarrow x<-1 \text{ یا } x>2 \\ x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \text{ یا } x \leq -1 \\ \sqrt{x^2-1}+1 \neq 0 \Rightarrow \text{همواره برقرار است} \end{cases}$$

$$\overset{\cap}{\Longrightarrow} \underline{D_f = (-\infty,-1) \cup (2,+\infty)}$$

۳)

$$2\log_x^a + \log_a^{\sqrt{x}} = 2 \overset{x=9}{\Longrightarrow} 2\log_9^a + \log_a^{\sqrt{9}} = 2 \Rightarrow \log_3^a + \log_a^3 = 2$$

$$\overset{\log_3^a = t}{\Longrightarrow} t + \frac{1}{t} = 2 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1 = \log_3^a \Rightarrow \boxed{a=3}$$

۴)

$$(\log^{\frac{5}{2}})x^2 + (\log^9)x - \log^{15} = 0 \qquad \log^5 = 1 - \log^2 = 1 - 0.3 = 0.7$$

$$\log^{\frac{5}{2}} + \underbrace{\log^9 - \log^{15}}_{\log^{\frac{9}{15}}} = \log^{\frac{5}{2}\times\frac{9}{15}} = \log^1 = 0 \xrightarrow[\text{صفر شد}]{\text{مجموع ضرایب}} \begin{cases} x_1 = 1 \\ x_2 = \dfrac{c}{a} = \dfrac{-\log^{15}}{\log^{\frac{5}{2}}} = -\dfrac{\log^5 + \log^3}{\log^5 - \log^2} \end{cases}$$

$$= -\frac{0.7 + 0.4}{0.7 - 0.3} = -\frac{1.1}{0.4} = -\frac{11}{4}$$

$$|x_1 - x_2| = |1 - (-\tfrac{11}{4})| = |1 + \tfrac{11}{4}| = |\tfrac{15}{4}| = \boxed{\tfrac{15}{4}}$$

۵)

$$\log_2^7 = 2.8$$

$$\log_5^2 = 0.5 \Rightarrow \log_2^5 = 2 \qquad \log_2^5 = \frac{1}{\log_5^2 = \frac{1}{2}} = 2$$

$$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{1+\log_2^5}{1+\log_2^7} = \frac{1+2}{1+2.8} = \frac{3}{3.8} = \frac{30}{38} = \underline{\frac{15}{19}}$$

۶)

$$\log_3^5 = 1.5 \qquad \log_3^2 = \frac{1}{\log_2^3} = \frac{1}{1.6}$$

$$\log_2^3 = 1.6 \Rightarrow \log_3^2 = \frac{1}{1.6} = \frac{10}{16} = \frac{5}{8}$$

$$\log_{15}^6 = \frac{\log_3^6}{\log_3^{15}} = \frac{1+\log_3^2}{1+\log_3^5} = \frac{1+\frac{5}{8}}{1+1.5} = \frac{\frac{13}{8}}{2.5} = \underline{\frac{13}{20}}$$

۷)

$$\left.\begin{array}{l}\log_8^{18} = m \\ \log_8^{18} = \dfrac{\log_2^{18}}{\log_2^8} = \dfrac{1+2\log_2^3}{3}\end{array}\right\} \Rightarrow \frac{2\log_2^3+1}{3} = m \Rightarrow \log_2^3 = \frac{3m-1}{2} \quad (I)$$

$$\log_4^{12} = \frac{\log_2^{12}}{\log_2^4} = \frac{2+\log_2^3}{2} \overset{(I)}{=} \frac{2+\frac{3m-1}{2}}{2} \overset{\times 2}{=} \frac{4+3m-1}{4} = \underline{\frac{3m+3}{4}}$$

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۸) $(0/4)^{2x-1} = (\frac{125}{8})^{x^2} \Rightarrow (\frac{4}{10})^{2x-1} = (\frac{5}{2})^{3x^2} \Rightarrow (\frac{2}{5})^{2x-1} = (\frac{5}{2})^{3x^2}$

$\Rightarrow (\frac{5}{2})^{-2x+1} = (\frac{5}{2})^{3x^2} \Rightarrow -2x+1 = 3x^2 \Rightarrow 3x^2+2x-1=0 \xrightarrow[a+c=b]{} \begin{cases} x_1=-1 & \text{غ ق ق} \\ x_2 = -\frac{c}{a} = \boxed{\frac{1}{3}} \end{cases}$

با توجه به دامنه‌ی عبارت لگاریتمی ($x>-\frac{1}{9}$)

$\log_8^{(9x+1)} = \log_8^4 = \log_{2^3}^{2^2} = \frac{2}{3}\cancel{\log_2^2} = \boxed{\frac{2}{3}}$

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۹) $\left.\begin{aligned} \log_2^3 &= a \\ \log_8^b &= \frac{2}{3}(1+a) \end{aligned}\right\} \Rightarrow \log_8^b = \frac{2}{3}(1+\log_2^3) = \frac{2}{3}(\log_2^2+\log_2^3) = \frac{2}{3}\log_2^6 = \log_{2^3}^{6^2}$

$\Rightarrow \log_8^b = \log_8^{36} \Rightarrow b=36$

$\log^{(3b-8)} = \log^{(108-8)} = \log^{100} = \boxed{2}$

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۱۰) (α و β ریشه‌های معادله‌ی درجه دوم هستند)

$\left.\begin{aligned} \frac{1}{\alpha+\beta} &= \frac{1}{S} = \log^4 \Rightarrow S = \log_4^{10} \\ S &= \frac{-b}{-4a} = \frac{b}{4a} \end{aligned}\right\} \Rightarrow \frac{b}{4a} = \log_4^{10} \Rightarrow \frac{4a}{b} = \log^4 \Rightarrow \frac{a}{b} = \frac{1}{4}\log^4$

$b+c = 2a \Rightarrow c = 2a-b$

$\Rightarrow \frac{a}{b} = \frac{1}{2}\log^2 \Rightarrow a = (\frac{1}{2}\log^2)b \quad (I)$

$\xrightarrow{(I)} c = \log^2 b - b = b(\log^2-1) = -b(1-\log^2) = -b(\log^{10}-\log^2) = (-\log^5)b \quad (II)$

$\frac{c}{a} \overset{(I),(II)}{=} \frac{-\log^5 \cancel{b}}{\frac{1}{2}\log^2 \cancel{b}} = -2\log_2^5 \Rightarrow (\frac{1}{\sqrt[8]{2}})^{\frac{c}{a}} = ((2^{-\frac{1}{8}})^{-2})^{\log_2^5} = (2^{\frac{1}{4}})^{\log_2^5}$

$= 5^{\log_2^{2^{\frac{1}{4}}}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$