

① $\log_n^m = a \Rightarrow m = n^a$
 $\log_{mn}^m = b \Rightarrow \log_{n^{a+1}}^{n^{a+1}} = b \Rightarrow \frac{a+1}{a+1} \log_n^{n^{a+1}} = b$
 $\Rightarrow b = \frac{a+1}{a+1} \Rightarrow 1 < b < 2 \Rightarrow [b] = 1$ ✓

② $y = \sqrt{\frac{n}{\log \frac{n}{r}}} \rightarrow x > 0$
 $\log \frac{n}{r} \neq 0 \Rightarrow n \neq 1$
 $y = \frac{\log(n^2 - n - r)}{\sqrt{n^2 - 1} + 1}$
 $\begin{cases} n^2 - n - r > 0 \Rightarrow (n+1)(n-r) > 0 \Rightarrow n < -1 \vee n > r \\ n^2 - 1 > 0 \Rightarrow n^2 > 1 \Rightarrow n > 1 \vee n < -1 \\ \sqrt{n^2 - 1} + 1 \neq 0 \Rightarrow \text{مقامی برابری} \end{cases}$
 $D_f = (-\infty, -1) \cup (r, +\infty)$ ✓

③ $r \log_n^a + \log_a^{\sqrt{n}} = r \Rightarrow r \log_a^a + \log_a^{\sqrt{n}} = r \Rightarrow \log_n^a + \log_a^r = r$
 $\log_n^a = t \Rightarrow t + \frac{1}{t} = r \Rightarrow t^2 - rt + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1 = \log_n^a \Rightarrow \boxed{a = n}$ ✓

④ $(\log^{\frac{1}{r}})^n + (\log^{\frac{1}{r}})^n - \log^{\frac{1}{r}} = 0$
 $\log^{\frac{1}{r}} + \log^{\frac{1}{r}} - \log^{\frac{1}{r}} = \log^{\frac{1}{r}} = 0$
 $\Rightarrow -\frac{0.16 + 0.16}{0.16 - 0.16} = -\frac{1.1}{0.16} = -\frac{11}{16}$
 $|n_1 - n_r| = |1 - (-\frac{11}{16})| = |1 + \frac{11}{16}| = |\frac{27}{16}| = \frac{27}{16}$ ✓

⑤ $\log_r^r = r, 1$
 $\log_r^r = 0, d \Rightarrow \log_r^d = r$
 $\log_{1/r}^{1/r} = \frac{\log_r^{1/r}}{\log_r^{1/r}} = \frac{1 + \log_r^d}{1 + \log_r^r} = \frac{1 + r}{1 + r, 1} = \frac{r}{r, 1} = \frac{r_0}{r_1} = \frac{1d}{19}$ ✓

⑥ $\log_n^d = 1, d$
 $\log_n^r = 1, r \Rightarrow \log_n^r = \frac{1}{1/r} = \frac{1}{r} = \frac{d}{n}$
 $\log_{1/d}^{1/d} = \frac{\log_n^{1/d}}{\log_n^{1/d}} = \frac{1 + \log_n^r}{1 + \log_n^d} = \frac{1 + \frac{d}{n}}{1 + 1/d} = \frac{\frac{1}{n}}{\frac{1}{d}} = \frac{1d}{n}$ ✓

⑦ $\log_n^m = m$
 $\log_n^m = \log_n^m = \frac{1 + r \log_n^m}{r}$
 $\log_{1/r}^{1/r} = \frac{\log_n^{1/r}}{\log_n^{1/r}} = \frac{r + \log_n^r}{r} = \frac{r + \frac{r(m-1)}{r}}{r} = \frac{r + m - 1}{r} = \frac{r + m - 1}{r}$ ✓

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$$\textcircled{1} (0/r)^{rn-1} = (1/r)^{rn} \Rightarrow (r/r)^{rn-1} = (r/r)^{rn} \Rightarrow (r/r)^{rn-1} = (r/r)^{rn} \Rightarrow (r/r)^{rn-1} = (r/r)^{rn}$$

$$\Rightarrow (r/r)^{rn-1} = (r/r)^{rn} \Rightarrow -rn+1 = rn \Rightarrow rn+rn-1=0 \Rightarrow \begin{cases} n_1 = -1 \\ n_2 = -\frac{1}{r} = \frac{1}{r} \end{cases} \textcircled{2}$$

باقی به دامنه عبارت گذاریم $(n > -\frac{1}{r})$

$$\log_a^{(rn+1)} = \log_a^r = \log_{r^r}^r = \frac{r}{r} \log_r^r = \frac{r}{r} \log_r^r = \frac{r}{r} \log_r^r$$

$$\textcircled{9} \left. \begin{aligned} \log_r^r &= a \\ \log_a^b &= \frac{r}{r}(1+a) \end{aligned} \right\} \Rightarrow \log_a^b = \frac{r}{r}(1+\log_r^r) = \frac{r}{r}(\log_r^r + \log_r^r) = \frac{r}{r} \log_r^r = \log_r^r$$

$$\Rightarrow \log_a^b = \log_a^r \Rightarrow b = r \textcircled{2}$$

$$\log^{(rb-1)} = \log^{(101-1)} = \log^{100} = \frac{1}{2} \checkmark$$

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$$\textcircled{10} \left. \begin{aligned} \frac{1}{\alpha+\beta} &= \frac{1}{5} = \log^r \Rightarrow 5 = \log^r \\ S &= \frac{b}{ra} = \frac{b}{ra} \end{aligned} \right\} \Rightarrow \frac{b}{ra} = \log^r \Rightarrow \frac{ra}{b} = \log^r \Rightarrow \frac{a}{b} = \frac{1}{r} \log^r \textcircled{2}$$

$$b+c=ra \Rightarrow c=ra-b \Rightarrow \frac{a}{b} = \frac{1}{r} \log^r \Rightarrow a = (\frac{1}{r} \log^r) b \quad (I)$$

$$\stackrel{(II)}{\Rightarrow} c = \log^r b - b = b(\log^r - 1) = -b(1 - \log^r) = -b(\log^r - \log^r) = (-\log^r) b \quad (II)$$

$$\frac{c}{a} \stackrel{(I),(II)}{=} \frac{-\log^r b}{\frac{1}{r} \log^r b} = -r \log^r \Rightarrow \left(\frac{1}{\sqrt{r}}\right)^{\frac{c}{a}} = ((r-1)-r) \log^r = (r-\frac{1}{r}) \log^r$$

$$= \log_r^r = \log^r = \frac{r}{r} = \frac{r}{r} \checkmark$$