«امیرعلی وحید» «گروه A»

۱-

$\log_n^m = a \qquad \log_{mn}^{m^2 n} = b$

$n^a = m \qquad m^b \times n^b = m^2 n \to n^{ab} \times n^b = n^{2a} \times n$

$n^{ab+b} = n^{2a+1} \longrightarrow ab+b = 2a+1$

$b = a+1 = 2a+1 \longrightarrow b = \frac{2a+1}{a+1} \Rightarrow b = \frac{a}{a+1}$

$[b] = 0$

۲-

$x = \sqrt{\frac{x}{\log_{\frac{1}{2}}^{x}}} \rightsquigarrow \frac{x}{\log_{\frac{1}{2}}^{x}} \geq 0 \longrightarrow x \geq 0 \quad \overset{\text{پس}}{\Longrightarrow} D_f = (0,+\infty) - \{1\}$

$\log_{\frac{1}{2}}^{x} \longrightarrow x > 0, x \neq 1$

$x = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$

$x^2 - x - 2 > 0 \longrightarrow (x-2)(x+1) > 0 \rightsquigarrow \underset{+\;0\;-\;0\;+}{-1 \quad 2} \rightsquigarrow (-\infty,-1) \cup (2,+\infty)$

$x^2 - 1 \geq 0 \rightsquigarrow (x-1)(x+1) \geq 0 \longrightarrow \underset{+\;0\;-\;0\;+}{} \longrightarrow [1,+\infty) \cup (-\infty,-1]$

$\cap \longrightarrow D_f = (-\infty,-1) \cup (2,+\infty)$

$\sqrt{x^2-1} + 1 \neq 0 \longrightarrow \sqrt{x^2-1} \neq -1$ ✓

۳-

$2\log_{3^2}^{a} + \log_a^3 = 2 \longrightarrow \log_3^a + \log_a^3 = 2 \longrightarrow \frac{1}{x} + x = 2 \longrightarrow 1 + x^2 = 2x \to x^2 - 2x + 1 = 0$

$\log_3^a = \frac{1}{\log_a^3}$

$(x-1)^2 \Rightarrow x = 1 \xrightarrow{\text{پس}} \log_a^3 = 1 \longrightarrow a = 3$

۴-

$\log 3 \simeq 0.48$

$\log 2 \simeq 0.3$

$(\log_{10}^5 - \log_{10}^{30})x^2 + (2\log_{10}^3)x - \log^{3\times 5}$

$2(0.4)x = 0.8x$

$\longrightarrow -(\log_{10}^3 + \log_{10}^{\cancel{10}/2}) \Longrightarrow -1.1$

$\Rightarrow \log_{10}^{\frac{10}{2}} - 0.48 \longrightarrow \cancel{\log_{10}^{10}} - \cancel{\log_{10}^{2}} - 0.48 = 0.3x^2$

$\xleftarrow{\text{پس}} 0.3x^2 + 0.8x - 1.1 = 0$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{0.64 - 4(-0.33)1.32}}{0.3} = \frac{1.4}{0.3} = \frac{14}{3}$

$\frac{14}{3}$

۵-

$\log_2^7 = 2.8$

$\log_5^2 = 0.5 \longrightarrow \frac{1}{\log_2^5} = 0.5 \longrightarrow \log_2^5 = 2$

$\log_{14}^{10} \to \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^{2\times 5}}{\log_2^{2\times 7}} \Rightarrow = \frac{\cancel{\log_2^2} + \log_2^5}{\cancel{\log_2^2} + \log_2^7} = \frac{3}{3.8}$

۶-

$\log_3^5 = 1.5$

$\log_2^3 = 1.6 \longrightarrow \frac{1}{\log_3^2} = 1.6 \longrightarrow \log_3^2 = 0.625$

$\log_{15}^6 = ? \longrightarrow \frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2 + \log_3^3}{\log_3^3 + \log_3^5} \Rightarrow \frac{1.625}{2.5} = 0.65$

۷-

$\log_8^{18} = m \longrightarrow \frac{\log_2^{18}}{\log_2^{8}} = \frac{\log_2^9 + \log_2^2}{3} = \frac{2\log_2^3 + 1}{3} = m$

$\hookrightarrow 2\log_2^3 + 1 = 3m$

$\log_4^{12} = ?$

$\hookrightarrow \frac{\log_2^{12}}{\log_2^{4}} = \frac{\log_2^4 + \log_2^3}{2} = \frac{2 + \log_2^3}{2}$

$\frac{3m-1}{2} = \log_2^3$

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$\log_4^{12} = \frac{3+3m}{4} \Longleftarrow \frac{2 + \frac{3m-1}{2}}{2} = \frac{\frac{4-1+3m}{2}}{2}$

۸-

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \longrightarrow \left(\frac{4}{10}\right)^{2x-1} \Rightarrow 2x - 1 = -3x^2$

$\left(\frac{5}{2}\right)^{3x^2} \to \left(\frac{2}{5}\right)^{-3x^2}$

$3x^2 + 2x - 1 = 0$

$x^2 + 2x - 3$

$(x+3)(x-1)$

$x = -\frac{3}{3}, \frac{1}{3}$

$x = -1, \frac{1}{3}$

۹-

$\frac{۱}{۲}\log_{۲} b = \frac{۳}{۲}(۱+a)$

$\frac{۱}{۲}\log_{۲} b = \underbrace{\cancel{۱}}_{\log_{۲}۲} + \alpha \log_{۲} ۳ \longrightarrow \log_{۲} b^{\frac{۱}{۲}} = \log_{۲} ۲ + \log_{۲} ۳ \Rightarrow b^{\frac{۱}{۲}} = \cancel{۲\times۳}^{\,۶} \longrightarrow \sqrt{b} = ۶$

$b = ۳۶ \longrightarrow \log ۳۶\times۳ - ۸ = ۱۰۰ \Rightarrow ۲$

۱۰-

$\frac{۴a}{b} = \log_{۱۰} ۴ \longrightarrow b = \frac{۴a}{\log_{۱۰} ۴} \longrightarrow a = \frac{b+c}{۲} \longrightarrow c = ۲a - b \longrightarrow c = ۲a - \frac{۴a}{\log_{۱۰} ۴} \longrightarrow \frac{c}{a} = (۲ - \frac{۴}{\log_{۱۰} ۴})$

$\frac{۱}{\sqrt[۸]{۲}} = ۲^{-\frac{۱}{۸}} \longrightarrow (۲^{-\frac{۱}{۸}})^{۲-\frac{۴}{\log ۴}} \longrightarrow ۲^{-\frac{۱}{۸}(۲-\frac{۴}{\log ۴})} \longrightarrow$

$\frac{۱}{\log_{۱۰} ۲} = \log_{۲} ۱۰ \longrightarrow ۲^{-\frac{۱}{۴}} \times ۲^{\frac{۱}{۲}\log_{۲} ۱۰}$

$۲^{\log_{۲} ۱۰} = ۱۰ \longrightarrow ۲^{-\frac{۱}{۴}} \times ۱۰^{\frac{۱}{۴}} \longrightarrow (\frac{۱۰}{۲})^{\frac{۱}{۴}} = \sqrt[۴]{۵}$