«امیرعلی وحید» «گروه A»

۱۸

-۱ (۱٫۵)

$\log_n^m = a \quad \log_{mn}^{m^2 n} = b$

$n^a = m$ ✓

$m^b \times n^b = m^2 n \rightarrow n^{ab} \times n^b = n^{2a} \times n$

$n^{ba+b} = n^{2a+1} \rightarrow ab + b = 2a + 1$ ✓ $(a+1)b = 2a+1$

$b = \frac{2a+1}{a+1} \Rightarrow b = \frac{a}{a+1}$

$[b] = 0$

$b = \frac{2a+1}{a+1}$

-۲ (۱٫۵)

$x = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \rightsquigarrow \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \rightarrow x \geq 0 \overset{\text{پس}}{\Rightarrow} D_f = (0, +\infty) - \{1\}$

$\log_{\frac{1}{2}} x \rightarrow x > 0, x \neq 1$

$x = \frac{\log_f (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightsquigarrow (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \rightsquigarrow (x-1)(x+1) \geq 0 \rightarrow [1, +\infty) \cup (-\infty, -1]$

$\cap \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$ ✓

$\sqrt{x^2-1} + 1 \neq 0 \rightarrow \sqrt{x^2-1} \neq -1$ ✓

-۳ (۲)

$2\log_{a^2}^{3} + \log_a^3 = 2 \rightarrow \frac{\log_a^3}{\dots} + \log_a^3 = 2 \rightarrow \frac{1}{x} + x = 2 \rightarrow 1 + x^2 = 2x \rightarrow x^2 - 2x + 1 = 0$

$(x-1)^2 \Rightarrow x = 1 \xrightarrow{\text{پس}} \log_a^3 = 1 \rightarrow a = 3$ ✓

-۴ (۲)

$\log 3 \simeq 0/48 \quad \log 2 \simeq 0/3$

$(\log_{10}^5 - \log_{10}^3)x^2 + (2\log_{10}^3)x - \log^{3 \times 5}$

$2(0/4)x = 0/8x$

$\Rightarrow \log_{10}^{\frac{10}{2}} - 0/4 \rightarrow \log_{10}^{10} - \log_{10}^{2} - 0/4 = 0/3x^2$

$-(\log_{10}^3 + \log_{10}^{\frac{10}{2}}) \quad 1 - \log_{10}^2 \quad 0/3$

$-1/1$

$0/3x^2 + 0/8x - 1/1 = 0$ (پس)

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{0/64 - 4(-0/33)1/32}}{0/3} = \frac{1/4}{0/3} = \frac{14}{3}$ ✓

-۵ (۲)

$\log_2^7 = 2/8$

$\log_5^2 = 0/5 \rightarrow \frac{1}{\log_2^5} = 0/5 \rightarrow \log_2^5 = 2$

$\log_{14}^{10} \rightarrow \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^{2 \times 5}}{\log_2^{2 \times 7}} = \frac{\log_2^2 + \log_2^5}{\log_2^2 + \log_2^7} = \frac{3}{3/8}$ ✓

-۶ (۲)

$\log_3^5 = 1/5$

$\log_2^3 = 1/6 \rightarrow \frac{1}{\log_3^2} = 1/6 \rightarrow \log_3^2 = 0/625$

$\log_{15}^6 = ? \rightarrow \frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2 + \log_3^3}{\log_3^3 + \log_3^5} \Rightarrow \frac{1/625}{2/5} = 0/65$ ✓

-۷ (۲)

$\log_8^{18} = m \rightarrow \frac{\log_2^{18}}{\log_2^{8}} = \frac{\log_2^9 + \log_2^2}{3} = \frac{2\log_2^3 + 1}{3} = m$

$2\log_2^3 + 1 = 3m$

$\log_4^{12} = ?$

$\frac{\log_2^{12}}{\log_2^{4}} = \frac{\log_2^4 + \log_2^3}{2} = \frac{2 + \log_2^3}{2} \qquad \frac{3m-1}{2} = \log_2^3$ (جایگزینی)

$\log_4^{12} = \frac{3 + 3m}{4}$ ✓ $\Leftarrow \frac{2 + \frac{3m-1}{2}}{2} = \frac{4 - 1 + 3m}{\frac{2}{2}}$

-۸ (۱)

$(\frac{4}{10})^{2x-1} = (\frac{5}{2})^{3x^2} \rightarrow (\frac{4}{10})^{2x-1} \Rightarrow 2x - 1 = -3x^2$

$(\frac{5}{2})^{3x^2} \rightarrow (\frac{2}{5})^{-3x^2} \qquad 3x^2 + 2x - 1 = 0$

$(x+3)(x-1) \qquad x^2 + 2x - 3$

$x = -3, \frac{1}{3}$

$x = -1, \frac{1}{3}$ ✓

$9x + 3 < 0$ غ ق ق

$\log_8^{9x+1} = \log_8^{9(\frac{1}{3})+1} = \log_8^4 = \frac{2}{3}$

-۹ (۲)

$\frac{1}{2}\log_{\sqrt{2}}^{b} = \frac{2}{\alpha}(1+\alpha)$

$\frac{1}{2}\log_{\sqrt{2}}^{b} = 1+\alpha\log_{2}^{2} \rightarrow \log b^{\frac{1}{2}} = \log_{2}^{2} + \log_{2}^{3} \Rightarrow b^{\frac{1}{2}} = 2\times 3 \rightarrow \sqrt{b} = 6$

$b = 36 \rightarrow \log^{36\times 3} - 8 = 100 \Rightarrow$ (۴) ✓

-۱۰ (۲)

$\frac{2a}{b} = \log_{10}^{2} \rightarrow b = \frac{2a}{\log_{10}^{2}} \rightarrow a = \frac{b+c}{2} \rightarrow c = 2a - b \rightarrow c = 2a - \frac{2a}{\log_{10}^{2}} \rightarrow \frac{c}{a} = \left(2 - \frac{2}{\log_{10}^{2}}\right)$

$2^{-\frac{1}{2}\times 2^{\frac{1}{2}\log_{2}^{10}}} \leftarrow \frac{1}{\log_{10}^{2}} = \log_{2}^{10} \leftarrow 2^{\frac{1}{8}\left(2 - \frac{2}{\log 2}\right)} \leftarrow \left(2^{-\frac{1}{8}}\right)^{2 - \frac{2}{\log 2}} \leftarrow \frac{1}{\sqrt{2}} = 2^{-\frac{1}{8}}$

$\hookrightarrow 2^{\log_{2}^{10}} = 10 \rightarrow 2^{-\frac{1}{2}\times 10^{\frac{1}{2}}} \rightarrow \left(\frac{1}{2}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{2}}$ ✓

-۱

$m = n^{a}$

$\log_{mn}^{m^2 n} = b \rightarrow \log_{n^{a+1}}^{n^{2a+1}} = b \rightarrow \frac{2a+1}{a+1} = b$

$b = \frac{2a+1+1-1}{a+1} = 2 - \frac{1}{a+1} \xrightarrow{a>0} [b] = 1$

-۲

الف) $\frac{n}{\log_{\frac{1}{2}}^{n}} \geq 0 \rightarrow \frac{n}{-\log_{2}^{n}} \geq 0 \xrightarrow{n>0} \log_{2}^{n} < 0$

$n < 2^{0} \rightarrow n < 1$

$(0, 1)$