۱- $\log_{mn}^{m} = \frac{\log_n^m}{\log_n^{mn}} = \frac{a}{\log_n^n + \log_n^m} = \frac{a}{1+a}$

$\log_{mn}^{m^2 n} = \log_{mn}^{mn} + \log_{mn}^{m} = 1 + \frac{a}{1+a} \Rightarrow \left[1 + \frac{a}{1+a}\right] = 1$

۲- الف) $x > 0$ $\quad \frac{x}{\log_{\frac{1}{2}}^x} \geq 0$ $\quad \log_{\frac{1}{2}}^x > 0$ $\quad x < 1$

$D_f = (0,1)$

ب) $x^2 - x - 2 > 0$ $\quad x^2 - 1 > 0$ $\quad D_f = (-\infty, -1) \cup (2, +\infty)$

$-1 \quad 2$

$+ \; | \; - \; | \; +$

$x^2 > 1$

$x > 1$

$x < -1$

۳- $2\log_9^a + \log_a^3 = 2$

$\log_3^a + \log_a^2 = 2$

$t + \frac{1}{t} = 2$

$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$

$t = 1 \quad \log_3^a = 1 \quad \boxed{a = 3}$

۴-

$\log\frac{5}{2} = \log 10 - \log 2 - \log 3 = 1 - 0{,}7 = 0{,}3$

$\log 9 = 2\log 3 = 0{,}8$

$\log 12 = \log 10 + \log 3 - \log 2 = 1 + 0{,}4 - 0{,}3 = 1{,}1$

$0{,}3x^2 + 0{,}8x - 1{,}1 = 0$

$x = 1 \quad x = -\frac{11}{3}$ $\qquad 1 + \frac{11}{2} = \boxed{\frac{13}{2}}$

۵- $\log_{16}^{10} = \frac{\log_2^{10}}{\log_2^{16}} = \frac{\log_2^2 + \log_2^5}{\log_2^2 + \log_2^7} = \frac{1+2}{1+2{,}8} = \frac{3}{3{,}8} = \frac{30}{38} = \frac{15}{19}$

۶- $\log_{15}^{4} = \frac{\log_3^4}{\log_3^{15}} = \frac{\log_3^3 + \log_3^3}{\log_3^3 + \log_3^5} = \frac{1 + \frac{5}{3}}{1 + \frac{5}{3}} = \frac{\frac{13}{3}}{\frac{5}{2}} = \frac{13}{20}$

۷- $\log_4^{12} = \frac{\log_8^{12}}{\log_8^4} = \frac{\log_8^4 + \log_8^3}{\frac{2}{3}} = \frac{\frac{2}{3} + \frac{3m-1}{4}}{\frac{2}{3}} = \frac{\frac{3m+3}{4}}{\frac{2}{3}} = \frac{3m+3}{4}$

$\log_8^{18} = \log_8^2 + \log_8^{3^2} = \frac{1}{3} + 2\log_8^3 = m$

$\log_8^3 = \frac{m - \frac{1}{3}}{2}$

۸- $\left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۲}{۵}\right)^{-۳x^۲}$

$-۱۲x^۲ = ۲x-۱$ $\qquad \log_۸^۴ = \frac{۲}{۳}$

$۳x^۲+۲x-۱=۰$

$x=-۱ \quad x=\frac{۱}{۳}$

$۹x+۱<۰$

۹- $\log_۸^b = \frac{۲}{۳}\left(۱+\log_۲^۴\right) = \frac{۲}{۳}\left(\log_۲^۲+\log_۲^۲\right) = \frac{۲}{۳}\left(\log_۲^۶\right) = \log_۸^{۶^۲}$

$\Rightarrow \log_۸^b = \log_۸^{۶^۲} \qquad \log(۳۷\times۳-۸) = \log۱۰۰ = ۲$

$b = ۶^۲$

۱۰- $\frac{۱}{\alpha}+\frac{۱}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{b}{۳}}{\frac{c}{-۸}} = \frac{-۲b}{c} = \log۴$

$۲a = b+c$ $\qquad \frac{-۴a+۲c}{c} = \log۴$

$\frac{-۴a}{c} = \log۴-۲$

$\frac{۴a}{c} = ۲-\log۴ = \log۱۰۰-\log۴ = \log۲۵ \Rightarrow \frac{a}{c} = \frac{۱}{۴}\log۲۵$