۱۸,۵

۱- (۲)

$$\log_{mn}^{m} = \frac{\log_{n}^{m}}{\log_{n}^{mn}} = \frac{a}{\log_{n}^{n}+\log_{n}^{m}} = \frac{a}{1+a}$$

$$\log_{mn}^{m^2 n} = \log_{mn}^{mn} + \log_{mn}^{m} = 1 + \frac{a}{1+a} \Rightarrow \left[1+\frac{a}{1+a}\right] = 1 \checkmark$$

۲- (۲)

الف) $x>0$

$$\frac{x}{\log_{\frac{1}{2}}^{x}} \geq 0 \quad \log_{\frac{1}{2}}^{x} > 0 \quad x<1$$

$D_f = (0,1)$ ✓

ب) $x^2-x-2>0$ $\quad x^2-1>0$ $\quad D_f = (-\infty,-1)\cup(2,+\infty)$ ✓

	-1		2	
	$+$	$\|$	$-$	$\|$
	$+$	$\|$	$+$	$\|$

$x^2>1$

$x>1$

$x<-1$

۳- (۲)

$$2\log_{9}^{a} + \log_{a}^{3} = 2$$

$$\log_{3}^{a} + \log_{a}^{a^2} = 2$$

$$t + \frac{1}{t} = 2$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$t=1 \quad \log_{3}^{a} = 1 \quad \boxed{a=3}$ ✓

۴- (۲)

$$\log\frac{5}{2} = \log 10 - \log 2 - \log 3 = 1 - 0{,}7 = 0{,}3$$

$$\log 9 = 2\log 3 = 0{,}8$$

$$\log 12{,}5 = \log 10 + \log 3 - \log 2 = 1 + 0{,}4 - 0{,}3 = 1{,}1$$

$$0{,}3x^2 + 0{,}8x - 1{,}1 = 0$$

$x=1 \quad x = -\frac{11}{3}$ $\qquad 1+\frac{11}{3} = \boxed{\frac{14}{3}}$ ✓

۵- (۲)

$$\log_{16}^{10} = \frac{\log_{2}^{10}}{\log_{2}^{16}} = \frac{\log_{2}^{2}+\log_{2}^{5}}{\log_{2}^{2}+\log_{2}^{7}} = \frac{1+2}{1+2{,}8} = \frac{3}{3{,}8} = \frac{30}{38} = \frac{15}{19}$$ ✓

۶- (۲)

$$\log_{12}^{5} = \frac{\log_{3}^{5}}{\log_{3}^{15}} = \frac{\log_{3}^{3}+\log_{3}^{5}}{\log_{3}^{3}+\log_{3}^{5}} = \frac{1+\frac{5}{2}}{\frac{5}{2}} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{5}$$ ✓

۷- (۲)

$$\log_{4}^{12} = \frac{\log_{8}^{12}}{\log_{8}^{4}} = \frac{\log_{8}^{4}+\log_{8}^{3}}{\frac{2}{3}} = \frac{\frac{2}{3}+\frac{3m-1}{4}}{\frac{2}{3}} = \frac{\frac{3m+3}{4}}{\frac{2}{3}} = \frac{3m+3}{4}$$ ✓

$$\log_{8}^{18} = \log_{8}^{2} + \log_{8}^{3^2} = \frac{1}{3} + 2\log_{8}^{3} = m$$

$$\log_{8}^{3} = \frac{m-\frac{1}{3}}{2}$$

۸- (۲)

$\left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۲}{۵}\right)^{-۲x^۲}$

$-۲x^۲ = ۲x-۱$ $\quad \log_۸^۴ = \frac{۲}{۳}$ ✓

$۲x^۲+۲x-۱=۰$

$x=-۱ \quad x=\frac{۱}{۲}$

قابل قبول

$۹x+۱<۰$

۹- (۲)

$\log_۸^b = \frac{۲}{۳}\left(۱+\log_۲^۳\right) = \frac{۲}{۳}\left(\log_۲^۲+\log_۲^۳\right) = \frac{۲}{۳}\left(\log_۲^۶\right) = \log_۸^{۶۴}$

$\Rightarrow \log_۸^b = \log_۸^{۶۴}$ $\quad \log(۳۷\times۳-۸) = \log۱۰۰ = ۲$ ✓

$b = ۶۴$

۱۰- (۵/۱)

$\frac{۱}{\alpha}+\frac{۱}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{b}{۲a}}{\frac{c}{-۸a}} = \frac{-۲b}{c} = \log۴$

$۲a = b+c$

$\frac{-۴a+۲c}{c} = \log۴$

$-\frac{۴a}{c} = \log۴-۲$

$\frac{۴a}{c} = ۲-\log۴ = \log۱۰۰-\log۴ = \log۲۵ \Rightarrow \frac{a}{c} = \frac{۱}{۴}\log۲۵$

$a = \frac{b+c}{۲} \rightarrow b = ۲a-c$

$\frac{۱}{x_۱+x_۲} = \frac{۱}{S} = \frac{۴a}{b} = \frac{۴a}{۲a-c} = \log۴ = ۲\log۲ \rightarrow \frac{۲a}{۲a-c} = \log۲$

معکوس $\rightarrow \frac{۲a-c}{۲a} = \log_۲^{۱۰} \rightarrow ۱-\log_۲^{۱۰} = \frac{c}{۲a} \rightarrow ۲-\log_۲^{۱۰۰} = \frac{c}{a}$

$\left(۲^{-\frac{۱}{۸}}\right)^{\frac{c}{a}} = \left(۲^{\frac{c}{a}}\right)^{-\frac{۱}{۸}} = \left(۲^{۲-\log_۲^{۱۰۰}}\right)^{-\frac{۱}{۸}} = \left(\frac{۴}{۱۰۰}\right)^{-\frac{۱}{۸}} = ۵^{\frac{۱}{۴}} = \boxed{\sqrt[۴]{۵}}$