۱۹٫۵

باسمه تعالی

نام و نام خانوادگی: علی ربانی — پاسخنامه تکلیف شماره: ۲۳ — کلاس یازدهم: A

۱- ۱ (۲)

$\log_n m = a \qquad n^a = m \qquad a > 0$

$\log_{mn} m^2 n = b \Rightarrow \log_{n^{a+1}} n^{2a+1} = b \Rightarrow \frac{2a+1}{a+1} \cancel{\log_n n} = b$

$\frac{2a+2-1}{a+1} = \frac{2(a+1)-1}{a+1} = 2 - \frac{1}{a+1} \Rightarrow b < 2 \qquad a > 0 \Rightarrow 2 > \frac{2a+1}{a+1} > 1 \Rightarrow \boxed{[b] = 1}$ ✓

۲- (۱٫۵)

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \Rightarrow \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \Rightarrow \log_{\frac{1}{2}} x > 0 \Rightarrow \cancel{x > 1}$ $\quad D = [1, +\infty)$

$x > 0$ (۱) $\qquad x < 1$ (۲)

چون مبنا بین صفر و یک جهت نامعادله رو عوض میکنه!

$\log_{\frac{1}{2}} x > 0, \neq 1$

①∩② → $(0, 1)$

ب) $y = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \qquad x^2 - 1 \geq 0$

$(x-2)(x+1) > 0 \qquad x^2 \geq 1$

-1		2	
$+$	$-$	$+$	

$x < -1$ یا $x > 2 \qquad x \geq 1$ یا $x \leq -1$

$\rightarrow D = (-\infty, -1) \cup (2, +\infty)$ ✓

۳- (۲)

$2 \log_x 9 + \log_a \sqrt{x} = 2 \qquad x = 9$

$\cancel{2 \log_9 a} + \log_a 3 = 2$

$\log_3 a + \log_a 3 = 2 \rightarrow \log_3 a + \frac{1}{\log_3 a} = 2 \qquad \log_3 a = 1 \Rightarrow \boxed{a = 3}$ ✓

$x + \frac{1}{x} = 2 \qquad \frac{x^2+1}{x} = 2 \qquad x^2 + 1 - 2x = 0$

$(x-1)^2 = 0 \Rightarrow x = 1$

۴- (۲)

$\log 2 \simeq 0{,}3$

$\log 3 \simeq 0{,}4$

$(\log \frac{5}{3}) x^2 + (\log 9) x - \cancel{\log 15} = 0 \qquad 0{,}3 x^2 + 0{,}8 x + 1{,}1 = 0$

$\cancel{\log 10} - \log 2 - \log 3 \qquad \cancel{\log 10} - \cancel{\log 2} + \log 3$

$1 - 0{,}3 - 0{,}4 = 0{,}3 \qquad 1 - 0{,}3 + 0{,}4 = 1{,}1$

$\log 9 = 2 \log 3 = 0{,}8$ $\quad$ اختلاف ریشه‌ها $= \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{0{,}64 - 1{,}32}}{|0{,}3|} = \frac{\sqrt{1{,}96}}{0{,}3} = \frac{1{,}4}{0{,}3} = \boxed{\frac{14}{3}}$ ✓

۵- ۳

$\log_2 7 = 2{,}8$

$\log_5 2 = 0{,}5$

$\log_{14} 10 = \frac{\log 10}{\log 14} = \frac{\log 5 + \log 2}{\log 14} = \frac{\log 5 + 2\log 2}{\log 2 + \log 7} = \frac{3\cancel{\log 2}}{2{,}8\log 2 + \cancel{\log 2}} = \frac{2}{2{,}8} = \frac{20}{28} = \boxed{\frac{15}{19}}$

$2{,}8 \log 2$

$\frac{\log 7}{\log 2} = 2{,}8 \qquad \frac{\log 2}{\log 5} = 0{,}5$

۶- ۲

$\log_3 5 = 1{,}5$

$\log_2 3 = 1{,}6$

$\log_{15} 6 = \frac{\log 6}{\log 15} = \frac{\log 2 + \log 3}{\log 5 + \log 3} = \frac{\frac{26}{16}\cancel{\log 3}}{\frac{25}{10}\cancel{\log 3}} = \frac{\cancel{26}^{13}}{\cancel{16}_{8}} \times \frac{\cancel{10}^{1}}{\cancel{25}_{5}} = \frac{13}{20}$

$= 0{,}65$

$\frac{\log 3}{\log 2} = 1{,}6 \qquad 10\log 3 = 16\log 2 \qquad \frac{10}{16}\log 3 = \log 2$

$\frac{\log 5}{\log 3} = \frac{15}{10} \qquad \log 5 = 1{,}5\log 3$

۷- ۲

$\log_8 18 = m$

$\log_{2^3} 3^2 \times 2 = m$

$\log_4 12 = \frac{1}{2}\log_2 2^2 \times 3 = \frac{1}{2}\log_{\cancel{2}} \cancel{2}^{\frac{3m+3}{2}} = \boxed{\frac{3m+3}{4}}$

$2^{3m} = 3^2 \times 2 \Rightarrow 2^{3m-1} = 3^2$

$2^{\frac{3m-1}{2}} = 3$

۸- ۲

$(0{,}4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \log_8 (9x+1)$

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{1000}{64}\right)^{x^2} \Rightarrow \left(\frac{4}{10}\right)^{2x-1} = \left(\frac{4}{10}\right)^{-3x^2} \Rightarrow 2x-1 = -3x^2$

$\left(\frac{10}{4}\right)^3$

$3x^2 + 2x - 1 = 0$

$x \to \frac{1}{3} \longrightarrow \log_8 9 \times \frac{1}{3} + 1 = \log_{2^3} 2^2 = \boxed{\frac{2}{3}}$

$x \to -1$ غ ق ق

۹-

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a) \qquad 8^{\frac{2}{3}(1+a)} = b \qquad \log_{10} 100 = \boxed{2}$

۲

$\log(4b-8) \qquad 2^{2+2a} = b$

$4b - 8 = 4 \times 27 = 108 - 8 = 100$ ① و $2^a = 3 \Rightarrow b = 2^2 \times 2^{2a} = b$

$4 \times 9 = 36$

۱۰-

۲

$-4ax^2 + bx + \frac{1}{2}c = 0$

$S = \frac{b}{-4a} \Rightarrow \frac{-4a}{b} = \log 4 \qquad b = \frac{\log 4}{-4a}$ و $a = -\frac{b\log 4}{4}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{\cancel{b}(\log 4 - 1)}{\frac{\cancel{b}\log 2}{2}}} = \qquad \cancel{b} + c = 2a$

$\left(2^{-\frac{1}{8}}\right)^{\frac{4(\log 4 - 1)}{\log 2}} = 2^{\frac{-(\log 4 - 1)}{4\log 2}} = \qquad \frac{\log 4}{-4a} + c = 2 \times \frac{b\log 4}{4}$

$c = 2 \times \frac{b\log 2}{2} - b = b\log 2 - b$

$2^{\frac{1-\log 4}{4\log 2}} = 2^{\frac{\log 10 - \log 4}{\log 16}} = 2^{\log_{16} 5} = 5^{\log_{16} 2} =$

دقت!

$5^{\frac{1}{4}\log_2 2} = 5^{\frac{1}{4}} = \sqrt[4]{5}$

$= \boxed{\cancel{\frac{1}{\sqrt[4]{5}}}}$