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تکلیف ۲۳ امین اسماعیلی

۱) ۲

$$\log_n m = a \Rightarrow n^a = m \Rightarrow \log_{mn} m^2 n = \log_{n^{a+1}} n^{2a+1} = \frac{2a+1}{a+1}\log_n n$$

$$\Rightarrow b = \frac{2a+1}{a+1},\ a>0 \Rightarrow b \in (1, \text{—}) \Rightarrow [b] = 1$$

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۲) الف: $x>0$ / $\log_{\frac{1}{2}} x = -\log_2 x \Rightarrow \dfrac{x}{-\log_2 x} \geq 0$ ریشه‌ها: $x=0$ ، $x=1$ ۲

$\log_2 x = 0 \Rightarrow x = 2^0 = 1$

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$$x>0 \cap x\in[0,1) \Rightarrow R_f = x \in (0,1)$$

ب: شرط‌ها:

$x^2 - x - 2 > 0 \to (x+1)(x-2)$ ؛ -1، 2: $+\ -\ +$ ؛ $x<-1 , x>2$

$x^2 - 1 > 0 \to x^2 > 1 \to x \in (-\infty,-1)\cup(1,+\infty)$

$\sqrt{x^2-1} + 1 \neq 0 \to$ به ازای هیچ x صفر نمی‌دهد

$$R_f : (-\infty,-1)\cup(2,+\infty)$$

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۳) $$2\log_9 a + \log_a \sqrt{9} = 2\log_9 a + \log_{(a)^2} (3)^2 = \log_9 a^2 + \frac{1}{\log_9 a^2} = 2$$ ۲

$$\Rightarrow \log_9 a^2 = 1 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

-3 غیرقابل قبول زیرا $2\log_9 a$ باید $a>0$ باشد

$4)\ \log_{10}^{\frac{5}{3}} = \log_{10}^{\cancel{5}\,\frac{10}{2}} - \log_{10}^{3} = \cancel{\left(\log_{10}^{10} - \log_{10}^{2}\right)}^{\overset{0.7}{1-0.3}} - \log_{10}^{3} = 0.3$

$\log_{10}^{9} = 2\log_{10}^{3} = 0.8 \qquad \log_{10}^{15} = \log_{10}^{5} + \log_{10}^{3} = 1.1 \quad (2)$

$0.3x^2 + 0.8x - 1.1 = 0 \xrightarrow[\text{صفر}]{\text{جمع ضرایب}} x = 1, -\frac{11}{3}$

$1 - \left(-\frac{11}{3}\right) = \frac{14}{3}$ ✓

$5)\ \log_{14}^{10} = \log_{14}^{2} + \log_{14}^{5} \quad , \quad \log_{14}^{2} = \frac{1}{\log_{2}^{14}} = \frac{1}{\log_{2}^{7} + \log_{2}^{2}} = \frac{1}{3.8}$

$\log_{14}^{5} = \frac{1}{\log_{5}^{14}} = \frac{1}{\underset{1.4}{\cancel{\log_{5}^{7}}} + \underset{0.5}{\cancel{\log_{5}^{2}}}} \rightsquigarrow \log_{5}^{7} = \frac{\overset{2.8}{\cancel{\log_{2}^{7}}}}{\underset{\frac{1}{0.5}=2}{\cancel{\log_{2}^{5}}}} = \frac{2.8}{2} = 1.4$

$\log_{14}^{5} = \frac{1}{1.9} = \frac{10}{19}$ (2)

$\log_{14}^{10} = \frac{1}{3.8} + \frac{1}{1.9} = \frac{3}{3.8} = \frac{15}{19}$ ✓

$6)\ \log_{15}^{6} = \log_{15}^{3} + \log_{15}^{2} \quad , \quad \log_{15}^{3} = \frac{1}{\log_{3}^{15}} = \frac{1}{\underset{1.5}{\cancel{\log_{3}^{5}}} + \underset{1}{\cancel{\log_{3}^{3}}}} = \frac{1}{2.5}$

$\log_{15}^{2} = \frac{1}{\log_{2}^{15}} = \frac{1}{\underset{\frac{12}{5}}{\cancel{\log_{2}^{5}}} + \underset{1.6}{\cancel{\log_{2}^{3}}}} \rightsquigarrow \log_{2}^{5} = \frac{\overset{1.5}{\cancel{\log_{3}^{5}}}}{\underset{\frac{1}{1.6}}{\cancel{\log_{3}^{2}}}} = \frac{1.5}{\frac{1}{1.6}} = \frac{12}{5}$

$\log_{15}^{2} = \frac{1}{4}$

(2)

$\log_{15}^{6} = \frac{1}{2.5} + \frac{1}{4} = \frac{13}{20} = 0.65$ ✓

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7) $\log_4 12 = \cancel{\log_4 4}^{\,1} + \log_4 3 \quad ; \quad \log_4 3 = \frac{1}{2}\log_2 3$ (2)

$\log_8 18 = \log_8 9 + \log_8 2 = \frac{2}{3}\log_2 3 + \frac{1}{\cancel{\log_2 8}^{\,3}} = m \Rightarrow \log_2 3 = \frac{3}{2}\left(m - \frac{1}{3}\right)$

$\Rightarrow \log_4 12 = 1 + \frac{1}{2}\left(\frac{3}{2}\left(m - \frac{1}{3}\right)\right) = \frac{3}{4}m + \frac{3}{4} = \boxed{\frac{3}{4}(m+1)}$ ✓

8) $(0.4)^{2x-1} = \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} = \left(\frac{2}{5}\right)^{-3x^2}$ (2)

$2x - 1 = -3x^2 \Rightarrow 3x^2 + 2x - 1 = 0 \begin{cases} x = -1 \\ x = \frac{1}{3} \end{cases}$ ✓

غ ق ق زیرا $9x+1<0$

$\log_8 (9 \times \frac{1}{3} + 1) = \log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}\log_2 2 = \frac{2}{3}$ ✓

9) $\frac{2}{3}(1+a) = \frac{2}{3}(\log_2 2 + \log_2 3) = \frac{2}{3}(\log_2 6) = \log_{2^3} 6^2 = \log_8 b$

$b = 36 \Rightarrow \log_{10} (3b - 8) = \log_{10} 100 = 2$ ✓ (2)

10) $-4ax^2 + bx + \frac{1}{2}c = 0 \xrightarrow{\text{عکس مجموع}} \frac{4a}{b} = \log_{10} 4 \quad ; \quad 2a = b + c$

$b = 2a - c$

$\frac{4a}{b} = \log_{10} 4 \rightarrow \frac{4a}{2a-c} = \log_{10} 4 \Rightarrow \frac{2a-c}{4a} = \log_4 10$ (2)

$\frac{c}{4a} = \frac{1}{2} - \log_4 10 \rightarrow \frac{c}{4a} = \log_4 2 - \log_4 10 = \log_4 0.2 \Rightarrow \frac{c}{a} = 4(\log_4 0.2)$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}} = 2^{-\frac{1}{8}\left(\frac{c}{a}\right)} = 2^{\frac{1}{2}(\log_4 0.2)} = \left(\frac{1}{\sqrt{5}}\right)^{-\frac{1}{2}} = \boxed{\sqrt[4]{5}}$ ✓

Celine