

$$m = n^a$$

$$\log_{mn}^{m^n} = \log_{mn}^{mn} + \log_{mn}^m = 1 + \log_{n^{a+1}}^{n^a} = 1 + \frac{a}{a+1} = b$$

$$\rightarrow 0 < \frac{a}{a+1} < 1 \Rightarrow 1 < b < 2 \Rightarrow [b] = 1$$

$$\text{الف) } \log_{\frac{1}{x}} x : x > 0$$

$$\sqrt{\frac{x}{\log_{\frac{1}{x}} x}} : \begin{cases} x \neq 1 \\ \frac{x}{\log_{\frac{1}{x}} x} \geq 0 \end{cases} \begin{matrix} \text{خرج مثبت} \\ \text{صورت مثبت} \end{matrix} \checkmark \Rightarrow D_f = (0, 1)$$

$$\text{ب) } \log_{x^2-x-2} (x^2-x-2) : x^2-x-2 > 0 \quad \frac{-1 \pm 2}{+ \phi - \phi +} \quad x < -1 \leq x > 2 \quad (I)$$

$$\sqrt{x^2-1} + 1 : x^2-1 \geq 0 \Rightarrow \frac{-1 \pm 1}{+ \phi - \phi +} \quad x < -1 \leq x > 1 \quad (II)$$

$$(I) \cap (II) \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$x \log_a^a + \log_a^x = 2 \Rightarrow x \log_{x^2}^a + \log_a^x = 2$$

$$\Rightarrow \log_{x^2}^a + \log_a^x - 2 = 0 \Rightarrow \log_{x^2}^a + \frac{1}{\log_{x^2}^a} - 2 = 0$$

$$\xrightarrow{\log_{x^2}^a = t} t + \frac{1}{t} - 2 = 0 \quad \times t \rightarrow t^2 - 2t + 1 = 0$$

$$\Rightarrow t = 1 \Rightarrow \log_{x^2}^a = 1 \Rightarrow a = x^2$$

$$\log^{\frac{a}{x}} = \log^{\frac{10}{x}} - \log^x = \log^{10} - \log^x - \log^x = 1 - \frac{1}{x^3} - \frac{1}{x^4} = \frac{1}{x^3}$$

$$\log^9 = 2 \log^x = 2 \times \frac{1}{x^4} = \frac{1}{x^4}$$

$$\log^{10} = \log^0 + \log^x = \underbrace{(\log^{10} - \log^x + \log^x)}_{\log^{\frac{10}{x}}} = 1 - \frac{1}{x^3} + \frac{1}{x^4} = \frac{1}{x^3}$$

$$\Rightarrow (\log^{\frac{10}{x}})^x + (\log^9)^x - (\log^{10})^x = \frac{1}{x^3} x^x + \frac{1}{x^4} x^x - \frac{1}{x^3} x^x = 0$$

$$\times 10 \rightarrow 3x^x + 1x^x - 11x^x = 0 \Rightarrow x = \frac{-11}{3} \quad 1 - (\frac{-11}{x}) = \frac{1}{x}$$

$$\log_{1/8}^{1/8} = \frac{\log_r^{1/8}}{\log_r^{1/8}} = \frac{\log_r^0 + \log_r^1}{\log_r^0 + \log_r^1} = \frac{\frac{1}{\log_r^0} + 1}{\log_r^0 + 1} = \frac{\frac{1}{0} + 1}{0 + 1} = \boxed{0/VA}$$

$$\log_{10}^4 = \frac{\log_r^4}{\log_r^{10}} = \frac{\log_r^1 + \log_r^3}{\log_r^2 + \log_r^8} = \frac{1}{\log_r^3 + 1} = \frac{\frac{1}{3} + 1}{1 + 1} = \boxed{0/40}$$

$$\log_{\Lambda}^{1\Lambda} = \log_{r^m}^{1\Lambda} = \frac{1}{r^m} \log_r^{1\Lambda} \Rightarrow \log_{r^m}^{1\Lambda} = r^m$$

$$\log_{r^m}^{1r} = \log_{r^m}^{1r} = \frac{1}{r^m} \log_r^{1r} = \frac{1}{r^m} (\log_r^r + \log_r^1) = \frac{1}{r^m} (r + \log_r^1) = 1 + \frac{1}{r^m} \log_r^1$$

$$\log_{r^m}^{1\Lambda} = \log_{r^m}^{r^m} + \log_{r^m}^1 = r \log_r^r + 1 \Rightarrow \log_r^r = \frac{r^m - 1}{r}$$

$$\Rightarrow \log_{r^m}^{1r} = 1 + \frac{1}{r^m} \log_r^r = 1 + \frac{r^m - 1}{r^m} = \boxed{\frac{r^m + r}{r}}$$

$$\left(\frac{0}{r}\right)^{r^{n-1}} = \left(\frac{r}{0}\right)^{r^{n-1}} = \left(\frac{0}{r}\right)^{1-r^n}$$

$$\left(\frac{1r0}{\Lambda}\right)^{r^n} = \left(\frac{0}{r}\right)^{r^n} = \left(\frac{0}{r}\right)^{1-r^n} = \left(\frac{0}{r}\right)^{r^n} \Rightarrow r^n = L r^n$$

$$\Rightarrow r^n + r^n - 1 = 0 \quad \frac{1}{r} \log_r^{r^{n+1}} = \log_{\Lambda}^r = \frac{r}{r} \log_r^r = \boxed{\frac{r}{r}}$$

$$r = r^a \quad b = 1^{\frac{r}{r}(1+a)} = r^{r(1+a)} = r^{r+r^a} = r^r \times r^{r^a} = r^r \times r^a \times r^a$$

$$= r \times r \times r = r^3 \quad \log_{r^3}^{r^6-1} = \log_{100} = \boxed{r}$$

$$\frac{ab}{ra} = \frac{1}{\log_r^r} \Rightarrow \log_r^r = \frac{ra}{b}$$

$$\frac{c}{a} = r - \frac{r}{\log_r^r} \quad \left(\frac{1}{\sqrt{r}}\right)^{\frac{c}{a}} = r^{-\frac{1}{\Lambda} \times \frac{c}{a}} = r^{-\frac{1}{\Lambda} \times (r - \frac{r}{\log_r^r})}$$

$$= r^{-\frac{1}{r}} + \frac{1}{r \log_r^r} = r^{\log_{1/4}^{1/4} - \frac{1}{r}} = r^{\log_{1/4}^{1/4} + \log_{1/4}^{\frac{1}{r}}} = r^{\log_{1/4}^0}$$

$$\log_{1/4}^{1/4} = \boxed{\Delta^{\frac{1}{r}} = \sqrt[5]{0}}$$