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$$m = n^a$$

-۱ (۲)

$$\log_{mn}^{m^n} = \log_{mn}^{mn} + \log_{mn}^m = 1 + \log_{n^{a+1}}^{n^a} = 1 + \frac{a}{a+1} = b$$

$$\rightarrow 0 < \frac{a}{a+1} < 1 \Rightarrow 1 < b < 2 \Rightarrow [b] = 1 \quad \checkmark$$

-۲ (۲)

$$\text{الف) } \log_{\frac{1}{x}} x : x > 0$$

$$\sqrt{\frac{x}{\log_{\frac{1}{x}} x}} : \begin{cases} x \neq 1 \\ \frac{x}{\log_{\frac{1}{x}} x} \geq 0 \end{cases} \begin{matrix} \text{خرج مثبت} \\ \text{صورت مثبت} \end{matrix} \checkmark \quad \Rightarrow D_f = (0, 1) \quad \checkmark$$

$x < 1 \Rightarrow$ صورت مثبت
 $x > 1 \Rightarrow$ صورت منفی $\times$

$$\text{ب) } \log_{x^2-x-2} (x^2-x-2) : x^2-x-2 > 0 \quad \frac{-1 \pm 2}{+ \phi - \phi +} \quad x < -1 \leq x > 2 \quad (I)$$

$$\sqrt{x^2-1} + 1 : x^2-1 \geq 0 \Rightarrow \frac{-1 \pm 1}{+ \phi - \phi +} \quad x < -1 \leq x > 1 \quad (II)$$

$$(I) \cap (II) \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty) \quad \checkmark$$

-۳ (۲)

$$x \log_a^a + \log_a^x = 2 \Rightarrow x \log_{x^2}^a + \log_a^x = 2$$

$$\Rightarrow \log_{x^2}^a + \log_a^x - 2 = 0 \Rightarrow \log_{x^2}^a + \frac{1}{\log_{x^2}^a} - 2 = 0$$

$$\xrightarrow{\log_{x^2}^a = t} t + \frac{1}{t} - 2 = 0 \quad \times t \rightarrow t^2 - 2t + 1 = 0$$

$$\Rightarrow t = 1 \Rightarrow \log_{x^2}^a = 1 \Rightarrow \boxed{a = x^2} \quad \checkmark$$

-۴ (۲)

$$\log^{\frac{9}{2}} = \log^{\frac{10}{2}} - \log^x = \log^{10} - \log^x - \log^x = 1 - \frac{1}{x^3} - \frac{1}{x^4} = \frac{1}{x^3}$$

$$\log^9 = 2 \log^x = 2 \times \frac{1}{x^4} = \frac{2}{x^4}$$

$$\log^{10} = \log^0 + \log^x = \underbrace{(\log^{10} - \log^x + \log^x)}_{\log^{\frac{10}{2}}} = 1 - \frac{1}{x^3} + \frac{1}{x^4} = \frac{1}{x^3}$$

$$\Rightarrow (\log^{\frac{9}{2}}) x^x + (\log^9) x - (\log^{10}) = \frac{1}{x^3} x^x + \frac{2}{x^4} x - \frac{1}{x^3} = 0$$

$$\times 10 \rightarrow 3x^x + 2x - 1 = 0 \Rightarrow x = \frac{-11}{3} \quad 1 - (-\frac{11}{3}) = \frac{14}{3} \quad \checkmark$$

$$\frac{ab}{ra} = \frac{1}{\log^f} \rightarrow \log^f = \frac{fa}{b}$$
$$\frac{c}{a} = r - \frac{r}{\log^f} \quad \left(\frac{1}{\sqrt[r]{r}}\right)^{\frac{r}{a}} = r^{-\frac{1}{r}} \times \frac{c}{a} = r^{-\frac{1}{r}} \times \left(r - \frac{r}{\log^f}\right)$$
$$= r^{-\frac{1}{f}} + \frac{1}{r \log^f} = r^{\log_{1/4}^{10} - \frac{1}{f}} = r^{\log_{1/4}^{10} + \log_{1/4}^{\frac{1}{f}}} = r^{\log_{1/4}^{\Delta}}$$
$$\Delta^{\log_{1/4}^r} = \boxed{\Delta^{\frac{1}{f}} = \sqrt[f]{\Delta}} \checkmark$$