نام و نام خانوادگی ...مهدی ابوفضلی... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...یازدهم ریاضی A...

۱

$\log_n m = a \longrightarrow n^a = m$

$\log_{mn} m^2 n = b \xrightarrow{n^a = m} \log_{n^a \cdot n} (n^a)^2 n = \log_{n^{a+1}} n^{2a+1} \longrightarrow \frac{2a+1}{a+1}\log_n n \longrightarrow \frac{2a+1}{a+1} = b$

$a > 0$

$[b] = ?$

$\Longrightarrow \frac{a+1+a}{a+1} = \frac{a}{a+1} + 1 = 1/\ldots \longrightarrow [1/\ldots] = 1$

۲

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \longrightarrow \frac{x}{\log_{\frac{1}{2}} x} \geq 0$ و مخرج $\neq 0$ $\longrightarrow x \neq 1$ (I)، ریشه‌ها $\longrightarrow x > 0$ (II)

$\Longrightarrow D_f = (0, 1)$

ب) $\frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \xrightarrow{\text{شرایط}} \begin{cases} x^2 - x - 2 > 0 \longrightarrow (x-2)(x+1) > 0 \\ x^2 - 1 \geq 0 \longrightarrow x^2 \geq 1 \longrightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \end{cases}$

اشتراک

$\Longrightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

۳

$2\log_x a + \log_a \sqrt{x} = 2 \longrightarrow 2\log_x a + \frac{1}{2}\log_a x = 2$

$x = 9$

$a = ?$

$\Longrightarrow \frac{2}{\log_a x} + \frac{1}{2}\log_a x = 2 \xrightarrow{\text{تغییر متغیر}} \log_a x = t \longrightarrow \frac{2}{t} + \frac{1}{2}t = 2$

$\longrightarrow \frac{1}{2}t^2 - 2t + 2 = 0$

$\xrightarrow{\times 2} t^2 - 4t + 4 = 0 \longrightarrow t = \frac{4 \pm \sqrt{16 - 16}}{2} = 2 \longrightarrow \begin{cases} x = 9 \\ \log_a x \end{cases} \longrightarrow \log_a x = 2$

$\Longrightarrow a^2 = 9 \longrightarrow a = 3$

۴

$\log 2 \simeq 0/3$

$\log 3 \simeq 0/4$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$

$? =$ اختلاف ریشه‌ها

اختلاف ریشه‌ها $= \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{b^2 - 4ac}}{|a|} = \frac{\sqrt{(2\log 3)^2 + 4(\log 5 - \log 3)(\log 5 + \log 3)}}{|\log 5 - \log 3|}$

مزدوج

$\log 9 = 2\log 3$

$\longrightarrow \begin{cases} \log 9 = 2\log 3 \\ \log 5/3 = \log 5 - \log 3 \end{cases}$

$\longrightarrow \frac{\sqrt{4(\log 3)^2 + 4((\log 5)^2 - (\log 3)^2)}}{|\log 5 - \log 3|}$

$\Longrightarrow \{ \log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0/3 = 0/7$

$\Longrightarrow$ ادامه در صفحه‌ی سوم

۵

$\log_2 7 = 2/8$

$\log_5 2 = 0/5$

$\log_{14} 10 = ?$

$\longrightarrow \log_{14} 10 = \frac{\log_c 2 + \log_c 5}{\log_c 2 + \log_c 7} = \frac{\log_c 10}{\log_c 14} = \log_{14} 10$

$\Longrightarrow c = 2 \longrightarrow \{ \frac{1 + \log_2 5}{1 + \log_2 7} = \frac{1 + \frac{1}{0/5}}{1 + 2/8} = \frac{3}{3/8} = \frac{\frac{3}{1}}{\frac{19}{5}} = \frac{3 \times 5}{19}$

$\Longrightarrow \frac{15}{19}$

* این سوال نیز مانند سوال ۵ حل می‌شود *

$\log_3^5 = 1.5$

$\log_2^3 = 1.6$

$\log_{15}^6 = ?$ $\longrightarrow \dfrac{\log_c^2+\log_c^3}{\log_c^5+\log_c^3} = \dfrac{\log_c^6}{\log_c^{15}} = \log_{15}^6 \xrightarrow{if} c=3 \longrightarrow \dfrac{\log_3^2+\log_3^3}{\log_3^5+\log_3^3} = \dfrac{\frac{1}{1.6}+1}{1.5+1}$ 6

$\Rightarrow \dfrac{\frac58+\frac88}{\frac32+\frac22} = \dfrac{\frac{13}{8}}{\frac52} = \dfrac{13\times \cancel{2}}{\cancel{8}_4\times 5} = \dfrac{13}{20} = 0.65$

$\log_8^{18} = m$

$\log_4^{12} = ? = x$ $\longrightarrow$ if $\log_4^{12} = x \longrightarrow 4^x = 12 \longrightarrow (2^2)^x = 2^2\times 3 \Rightarrow 2^{2x} = 2^2\times 3 \longrightarrow 3 = 2^{2x-2}$ 7

$8^m = 18 \longrightarrow 2^{3m} = 18 \xrightarrow{\div 2} 2^{3m-1} = 9 \Rightarrow \begin{cases} 3^2 = 2^{4x-4} \\ 9 = 2^{3m-1} \end{cases} \longrightarrow 4x-4 = 3m-1$

$\Rightarrow$ جواب مسئله $= x \Rightarrow 4x = 3m-1+4 \longrightarrow x = \dfrac{3m+3}{4} = \dfrac34(m+1)$

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \Rightarrow \left(\frac25\right)^{2x-1} = \left(\left(\frac52\right)^3\right)^{x^2} \longrightarrow \left(\frac25\right)^{2x-1} = \left(\left(\frac25\right)^{-1}\right)^{3x^2} \Rightarrow \left(\frac25\right)^{2x-1} = \left(\frac25\right)^{-3x^2}$

$\log_8^{(9x+1)} = ?$ $\Rightarrow 2x-1 = -3x^2 \longrightarrow 3x^2+2x-1=0 \longrightarrow x = \dfrac{-2\pm\sqrt{4+12}}{6} = \dfrac{-2\pm\sqrt{16}}{6}$ 8

$\Rightarrow x = \dfrac{-2\pm 4}{6}$ $\begin{cases} -1 & \text{غ ق ق} \\ \frac13 & ✓ \end{cases}$ $\longrightarrow \log_8^{(9x+1)} \xrightarrow{x=1/3} \log_8^4 = \log_{2^3}^{2^2}$

$\Rightarrow \frac23 \log_2^2 = \frac23$

$\log_2^3 = a$

$\log_8^b = \frac23(1+a)$ $\longrightarrow \log_{2^3}^b = \frac{1}{\cancel{3}}\log_2^b = \frac{2}{\cancel{3}}(1+a) \Rightarrow b = 2^{2(1+a)} = 2^{2+2a} = 2^{2a}\times 4$

$\log(3b-8) = ?$ $\Rightarrow 2^{2a} = 2^{2\log_2^3} \longrightarrow 2^{2a}\times 4 = 2^{\log_2^9}\times 4 = 9^{\log_2^2}\times 4 = 9\times 4 = 36$ 9

$\longrightarrow \log(3\times 36-8) = \log(108-8) = \log^{100} = 2$

$-4ax^2+bx+\frac12 c = 0$ $\longrightarrow \dfrac{1}{\frac{-b}{-4a}} = \dfrac{4a}{b} = \log^4 \xrightarrow{\div 4} \dfrac ab = \dfrac{\log^4}{4} = \dfrac{2\log^2}{4}$

$\dfrac{1}{\alpha+\beta} = \log^4$

$a = \dfrac{b+c}{2} \longrightarrow 2a = b+c \longrightarrow \dfrac{2a}{b} = 1+\dfrac cb \longrightarrow \dfrac{2a}{b} = \dfrac{\cancel{4}\log^2}{\cancel{4}} = 1+\dfrac cb$ 10

$\left(\frac{1}{\sqrt[4]{2}}\right)^{\frac ca} = ?$ $\begin{cases} \frac ab = \log^2/2 \\ \frac cb = \log^2 - 1 \end{cases} \longrightarrow \dfrac ca = \dfrac{\log^2-1}{\log^2/2} = \dfrac{\log^2-\log^{10}}{\frac12\log^2} = \dfrac{\log^{2/10}}{\log^{\sqrt2}} = \log_{\sqrt2}^{1/5}$

$\longrightarrow \left(\frac{1}{\sqrt[4]{2}}\right)^{\log_{\sqrt2}^{1/5}} \longrightarrow \left(\frac15\right)^{\log_{\sqrt2}^{\frac{1}{\sqrt[4]{2}}}} = \left(\frac15\right)^{\frac{-1/8}{1/2}\log_2^2} = \left(\frac15\right)^{-\frac14} = 5^{1/4} = \sqrt[4]{5}$

ادامه‌ی سوال 4

$$\begin{cases} \lg 2 = 0.3 \\ \lg 3 = 0.4 \\ \lg 5 = 0.7 \end{cases} \longrightarrow \frac{\sqrt{4\left(\cancel{(\lg 3)^2}\right) + (\lg 5)^2 - \cancel{(\lg 3)^2}}}{|\lg 5 - \lg 3|}$$

$$\Longrightarrow \frac{\sqrt{4\,(0.7)^2}}{|0.7 - 0.4|} = \frac{\sqrt{4 \times 0.49}}{0.3} = \frac{2 \times 0.7}{0.3} = \frac{1.4}{0.3} = \boxed{\frac{14}{3}}$$