نام و نام خانوادگی ....آرمین ایزدی.... پاسخنامه تشریحی تکلیف شماره ..۲۳.. کلاس ....A....

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$\log_n m = a \quad \log_{mn} m^2 n = b \quad n^a = m \rightarrow \log_{n^{a+1}} n^{2a+1} = b \rightarrow \frac{2a+1}{a+1} = b$

$b = \frac{2a+1}{a+1} = \frac{2a+2-1}{a+1} = 2 - \frac{1}{a+1} \Rightarrow 1 < b < 2 \Rightarrow [b] = 1$ (with $\frac{1}{a+1}$ marked < 0)

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الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$

$x > 0$

$\log_{\frac{1}{2}} x \neq 0 \rightarrow x \neq 1$

$\frac{x}{\log_{\frac{1}{2}} x} \geq 0$: $0<x<1 \rightarrow \log_{\frac{1}{2}} x > 0 \rightarrow x > 0$; $x > 1 \rightarrow \log_{\frac{1}{2}} x < 0 \rightarrow x < 0$

$\Rightarrow D = (0, 1)$

ب) $y = \log_{\sqrt{x^2-1}+1} (x^2 - x - 2)$

$\sqrt{x^2-1} \rightarrow x^2 - 1 \geq 0 \rightarrow x \geq 1, x \leq -1$

$\sqrt{x^2-1} + 1 \neq 0 \rightarrow \sqrt{x^2-1} \neq -1$ ✓

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x > 2, x < -1$

$\Rightarrow D = (-\infty, -1) \cup (2, +\infty)$

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$(\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0 \longrightarrow (\log 5 - \log 3) x^2 + (2\log 3) x - (\log 5 + 3) = 0$

$\longrightarrow (1 - 0.3 - 0.4) x^2 + 0.8x - (1 - 0.3 + 0.4) \longrightarrow 0.3x^2 + 0.8x + [illegible] = 0$ (1.1 written above the last term)

$\longrightarrow b^2 - 4ac \rightarrow 0.64 - 4(-1.1)(0.3) = 1.96 \quad x_1 = \frac{-0.8 + 1.4}{0.6} = 1, \; x_2 = \frac{-0.8 - 1.4}{0.6} = \frac{-1.1}{0.3}$

$x_1 - x_2 = 1 + \frac{1.1}{0.3} = 1 + \frac{11}{3} = \frac{14}{3}$

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$2\log_9 a + \log_a \sqrt{9} = 2 \rightarrow \log_3 a + \log_a 3 = 2 \rightarrow \frac{1}{\log_a 3} + \log_a 3 = 2 \rightarrow (\log_a 3)^2 - 2\log_a 3 + 1 = 0$

$\longrightarrow 4 - 4(1)(1) = 0 \Rightarrow \log_a 3 = 1 \rightarrow a = 3$

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$\log_2 7 = 2.8, \; \log_5 2 = 0.5 \Rightarrow 7 = 2^{2.8}, \; 2 = 5^{0.5} \Rightarrow 7 = (5^{0.5})^{2.8} = 5^{1.4}$

$10 = 2 \times 5 = 5^{0.5} \times 5 = 5^{1.5}, \; 14 = 2 \times 7 = 5^{0.5} \times 5^{1.4} = 5^{1.9}$

$\log_{14} 10 = \log_{5^{1.9}} 5^{1.5} = \frac{15}{19}$

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