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نام و نام خانوادگی ......آرمین......ایزدی...... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...A...

۱ (۲)

$\log_n m = a \qquad \log_{mn} m^2 n = b \qquad n^a = m \to \log_{n^{a+1}} n^{2a+1} = b \to \frac{2a+1}{a+1} = b$

$b = \frac{2a+1}{a+1} = \frac{2a+2-1}{a+1} = 2 - \frac{1}{a+1} \Rightarrow 1 < b < 2 \Rightarrow [b] = 1$ ✓

۲ (۳)

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ $\quad x > 0 \quad \log_{\frac{1}{2}} x \neq 0 \to x \neq 1 \quad \frac{x}{\log_{\frac{1}{2}} x} \geq 0$

$0 < x < 1 \to \log_{\frac{1}{2}} x > 0 \to x > 0$

$x > 1 \to \log_{\frac{1}{2}} x < 0 \to x < 0$

$\Rightarrow D = (0, 1)$ ✓

ب) $y = \log_{\sqrt{x^2-1}+1} (x^2-x-2) \to$

$\sqrt{x^2-1} \to x^2 - 1 \geq 0 \to x \geq 1 , x \leq -1$

$\sqrt{x^2-1} + 1 \neq 0 \to \sqrt{x^2-1} \neq -1$ ✓

$x^2 - x - 2 > 0 \to (x-2)(x+1) > 0 \to x > 2 , x < -1$

$\to D = (-\infty, -1) \cup (2, +\infty)$ ✓

۴ (۲)

$(\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0 \to (\log 5 - \log 3) x^2 + (2 \log 3) x - (\log 5 + 3) = 0$

$\to (1 - 0.3 - 0.4) x^2 + 0.8x - (1 - 0.3 + 0.4) \to 0.3 x^2 + 0.8x + 1.1 = 0$

$\to b^2 - 4ac \to 0.64 - 4(-1.1)(0.3) = 1.96 \qquad x_1 = \frac{-0.8 + 1.4}{0.6} = 1 , \; x_2 = \frac{-0.8 - 1.4}{0.6} = \frac{-1.1}{0.3}$

$x_1 - x_2 = 1 + \frac{1.1}{0.3} = 1 + \frac{11}{3} = \frac{14}{3}$ ✓

۳ (۲)

$2 \log_9 a + \log_a \sqrt{9} = 2 \to \log_3 a + \log_a 3 = 2 \to \frac{1}{\log_a 3} + \log_a 3 = 2 \to (\log_a 3)^2 - 2 \log_a 3 + 1 = 0$

$\to 4 - 4(1)(1) = 0 \Rightarrow \log_a 3 = 1 \to a = 3$ ✓

۵ (۲)

$\log_2 7 = 2.8 , \log_5 2 = 0.5 \Rightarrow 7 = 2^{2.8} , 2 = 5^{0.5} \Rightarrow 7 = (5^{0.5})^{2.8} = 5^{1.4}$

$10 = 2 \times 5 = 5^{0.5} \times 5 = 5^{1.5} , \; 14 = 2 \times 7 = 5^{0.5} \times 5^{1.4} = 5^{1.9}$

$\log_{14} 10 = \log_{5^{1.9}} 5^{1.5} = \frac{15}{19}$ ✓

۶ (۲)

$\log_3 5 = 1.5 \,,\, \log_2 3 = 1.6 \to 5 = 3^{1.5} \,,\, 3 = 2^{1.6} \Rightarrow 5 = (2^{1.6})^{1.5} = 2^{2.4}$

$6 = 2 \times 3 = 2 \times 2^{1.6} = 2^{2.6} \,,\, 15 = 3 \times 5 = 2^{1.6} \times 2^{2.4} = 2^4$

$\log_{2^4} 2^{2.6} = \frac{2.6}{4} = \boxed{0.65}$ ✓

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۷ (۲)

$\log_8 18 = m$

$8^m = 18 \to (2^3)^m = 2 \times 3^2 \to 3m = \log_2 2\times3^2 \to 3m = 1 + 2\log_2 3 \to \log_2 3 = \frac{3m-1}{2}$

$\log_4 12 = x \to 4^x = 12 \to (2^2)^x = 2^2 \times 3 \to 2x = \log_2 2^2\times3 = 2 + \log_2 3$

$x = \frac{2+\log_2 3}{2} = 1 + \frac{\log_2 3}{2} \to x = 1 + \frac{3m-1}{4} \to \boxed{x = \frac{3m+3}{4}}$ ✓

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۸ (۲)

$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \to (2 \times 5^{-1})^{2x-1} = (5^3 \times 2^{-3})^{x^2} \to 2^{2x-1} \times (5^{-1})^{2x-1} = 2^{2x-1} \times 5^{-(2x-1)}$

$(5^3)^{x^2} \times (2^{-3})^{x^2} = 5^{3x^2} \times 2^{-3x^2} \to 2^{2x-1} \times 5^{-(2x-1)} = 2^{-3x^2} \times 5^{3x^2}$

$2 \to 2x - 1 = -3x^2 \to 3x^2 + 2x - 1$

$5 \to 3x^2 + 2x = 1$

$\Longrightarrow x_1 = \frac{-2+4}{6} = \frac{1}{3} \,,\, x_2 = \frac{-2-4}{6} = -1 \,,\, \begin{cases} x = \frac{1}{3} \to \log_8 4 = \boxed{\frac{2}{3}} \\ x = -1 \Rightarrow \log_8 \ldots \text{ غ ق ق} \end{cases}$ ✓

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۹ (۰)

$\log_8 b = \frac{2}{3}(1+a) \to b = 8^{\frac{2}{3}(1+a)} \to 3b - 8 = 3 \times 8^{\frac{2}{3}(1+a)} - 8 \to 3b - 8 = 8(3 \times 8^{\frac{2}{3}(1+a)-1} - 1)$

$\to 3b - 8 = 8(3 \times 2^{2a-1} - 1) \quad \log 3b - 8$

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۱۰ (۰)

$a = \frac{b+c}{2} \to b = 2a - c$

$\frac{1}{x_1 + x_2} = \frac{1}{S} = \frac{4a}{b} = \frac{4a}{2a-c} = \lg 4 = 2\lg 2 \to \frac{2a}{2a-c} = \lg 2$

معکوس $\to \frac{2a-c}{2a} = \lg_2 10 \to 1 - \lg_2 10 = \frac{c}{2a} \to 2 - \lg_2 100 = \frac{c}{a}$

$(2^{-\frac{1}{8}})^{\frac{c}{a}} = (2^{\frac{c}{a}})^{-\frac{1}{8}} = (2^{2-\lg_2 100})^{-\frac{1}{8}} = \left(\frac{4}{100}\right)^{-\frac{1}{8}} = 5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$

۹-

$$\lg_{۸}^{b} = \frac{۱}{\cancel{۳}}\lg_{۲}^{b} = \frac{۲}{\cancel{۳}}\Big(\underbrace{\cancel{\overset{۱}{\lg_{۲}^{۲}} \overset{+}{+} \overset{a}{\lg_{۲}^{۳}}}}_{\lg_{۲}^{۶}}\Big) \rightarrow \lg_{۲}^{b} = ۲\lg_{۲}^{۶} = \lg_{۲}^{۶^{۲}}$$

$$b = ۳۶ \rightarrow \lg(۳b - ۸) = \lg ۱۰۰ = \boxed{۲}$$