نام و نام خانوادگی ...ابوالفضل ابوفضلی... پاسخنامه تشریحی تکلیف شماره ...۲... کلاس ....

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$2x^2+y^2-4x+6y+k=2x^2-4x+y^2+6y+k=0 \Rightarrow 2(x^2-2x)+y^2+6y+k=0$

$\xrightarrow{\text{مربع کامل}} \begin{cases} 2(x^2-2x) \longrightarrow 2((x-1)^2-1)=2(x-1)^2-2 \\ (y+3)^2-9 \end{cases} \Rightarrow 2(x-1)^2+(y-3)^2-9-2+k=0$

$\Rightarrow \overbrace{2(x-1)^2+(y+3)^2}^{\text{همواره مثبت}}=11-k \longrightarrow 11-k\geq 0 \Rightarrow k\leq 11$

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$f(x)=\begin{cases} x^2+4x \ ; & x\geq a \\ 2x+a+2 \ ; & x\leq a \end{cases} \xrightarrow{x=a} \begin{cases} a^2+4a \\ 2(a)+a+2 \end{cases} \Rightarrow a^2+4a=3a+2 \Rightarrow$

$a^2+a-2=0 \Rightarrow (a+2)(a-1)=0$

$\Rightarrow a=\begin{cases} +1 \\ -2 \end{cases} \xrightarrow{a>0} a=1 \Rightarrow f(1)=2(1)+1+2=5$

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$f(x)=\begin{cases} 2x^2-b \ ; & x\geq 1 \\ & x\leq -3 \\ 2x+a \ ; & -3\leq x\leq -1 \end{cases} \xrightarrow{x=-3} 2x^2-b=2x+a \Rightarrow 18-b=-6+a$

$\Rightarrow a+b=18+6=24$

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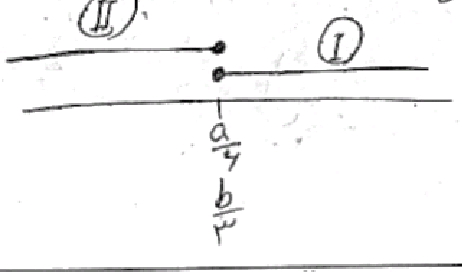
$y=\sqrt{6x-a}+\sqrt{b-3x} \Rightarrow \begin{cases} 6x-a\geq 0 \longrightarrow x\geq \frac{a}{6} \\ b-3x\geq 0 \longrightarrow x\leq \frac{b}{3} \end{cases} \xrightarrow{\text{I}\cap\text{II}} D_y=\{2\}$



$\Rightarrow \frac{a}{6}=\frac{b}{3}=2 \Rightarrow \begin{cases} a=12 \\ b=6 \end{cases} \Rightarrow \frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

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الف) $\begin{cases} \dfrac{x+1}{|x-3|}\geq 0 \xrightarrow{x+3} \begin{array}{c|c} - & + \\ \end{array}_{-1} \Rightarrow \{x \mid x\geq -1 ,\ x\neq 3\} \\ \dfrac{6-x}{x^2+2x+4}\geq 0 \longrightarrow \begin{array}{c|c} + & - \\ \end{array}_{6} \Rightarrow x\leq 6 \end{cases} \xrightarrow{\cap} D_f=[-1,6]-\{3\}$

بجز $x=3$ همواره مثبت

$a>0 ,\ \Delta<0 \Rightarrow$ همواره مثبت

ب) $\begin{cases} [x]-4\geq 0 \Rightarrow [x]\geq 4 \Rightarrow x\geq 4 \\ 2-[x]\geq 0 \Rightarrow 2\geq [x] \Rightarrow x<3 \end{cases} \xrightarrow{\text{اشتراک}} D_f=\varnothing$

الف) $\dfrac{x^2-x-6}{x^4-13x^2+36}\geq 0 \Rightarrow \dfrac{(x-3)(x+2)}{(x^2-9)(x^2-4)}=\dfrac{(x-3)(x+2)}{(x-3)(x-2)(x+3)(x+2)}\geq 0 \Rightarrow \begin{array}{ccccccccc} & -3 & & -2^* & & 2 & & 3^* & \\ + & | & - & | & - & | & + & | & + \end{array}$

$\Rightarrow \boxed{D_f=(-\infty,-3)\cup(+2,3)\cup(3,+\infty)}$ ۶

ب) $\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}\geq 0 \Rightarrow \dfrac{(x-1)\overbrace{(x^2-x-6)}^{(x-3)(x+2)}}{(x+1)\underbrace{(x^2+x-6)}_{(x+3)(x-2)}}\geq 0 \Rightarrow \begin{array}{ccccccccccccc} & -3 & & -2 & & -1 & & 1 & & 2 & & 3 & \\ + & | & - & | & + & | & - & | & + & | & - & | & + \end{array}$

$\Rightarrow D_f=(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

الف) $y=\sqrt{[-x]-2} \Rightarrow [-x]-2\geq 0 \Rightarrow [-x]\geq 2 \Rightarrow -x\geq 2 \Rightarrow x\leq -2 \Rightarrow$

$D_f=(-\infty,-2]$

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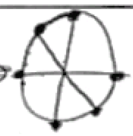
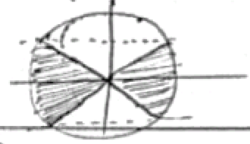
ب) $y=\dfrac{5x^2+6}{[x]^2-2[x]-3} \Rightarrow [x]^2-2[x]-3\neq 0 \Rightarrow ([x]-1)^2\neq 4 \Rightarrow \begin{cases}[x]-1\neq 2\\ [x]-1\neq -2\end{cases}$

$\Rightarrow \begin{cases}[x]\neq 3 \longrightarrow x\neq[3,4)\\ [x]\neq -1 \quad\quad [-1,0)\end{cases} \Rightarrow D_f=\mathbb{R}-\{[-1,0)\cup[3,4)\}$

الف) $y=\dfrac{\cot x+1}{\tan x+1} \Rightarrow \begin{cases}\sin x\neq 0,\ \cos x\neq 0\\ \tan x\neq -1\end{cases} \Rightarrow$  $\Rightarrow D_f=\mathbb{R}-\left\{\dfrac{k\pi}{2},\ \dfrac{3k\pi}{2}+\dfrac{\pi}{4}\right\}$

ب) $y=\sqrt{1-4\sin^2 x} \Rightarrow (1-2\sin x)(1+2\sin x)\geq 0 \Rightarrow \begin{array}{ccccc} & -\frac{1}{2} & & \frac{1}{2} & \\ - & | & + & | & - \end{array} \Rightarrow -\dfrac{1}{2}\leq \sin x\leq \dfrac{1}{2}$ ۸

 $\Rightarrow D_f=\left[\pi-\dfrac{\pi}{6},\ \pi+\dfrac{\pi}{6}\right]$

الف) $\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \Rightarrow 1-\log_{\frac{1}{2}}^{x-1}\geq 0 \Rightarrow \log_{\frac{1}{2}}^{x-1}\leq 1 \Rightarrow x-1\leq \frac{1}{2} \Rightarrow x\leq 1{,}5$

$x-1>0 \rightarrow x>1 \Rightarrow \boxed{D_f=[1,\ 1{,}5]}$

اشتراک: $(-\infty,-1)\cup(-1,0)\cup(3,+\infty)-\{4\}$ ۹

ب) $y=\dfrac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}} \Rightarrow \begin{cases} x(x-1)\geq 0 \quad \begin{array}{ccccc} & 0 & & 1 & \\ + & | & - & | & + \end{array} \\ \quad\quad\quad\quad\quad x^2-3x>0 \quad \begin{array}{ccccc} & 0 & & 3 & \\ + & | & - & | & + \end{array} \\ \log_4^{x^2-3x}\neq 1 \Rightarrow x^2-3x\neq 4 \Rightarrow x^2-3x-4\neq 0 \Rightarrow x\neq 4,-1 \end{cases}$

الف) $y=\sqrt{2^{4x-x^2}-8} \Rightarrow 2^{4x-x^2}\geq 8 \Rightarrow 4x-x^2\geq 3 \Rightarrow -x^2+4x-3\geq 0 \Rightarrow -(x-3)(x-1)\geq 0$

$\begin{array}{ccccc} & 1 & & 3 & \\ - & | & + & | & - \end{array} \Rightarrow D_f=[1,3]$

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ب) $\left(\dfrac{3x+5}{2x+4}\right)! \Rightarrow \dfrac{3x+5}{2x+4}\in W \Rightarrow x\in -\dfrac{4w-5}{2w-3}=\dfrac{4w-5}{3-2w}$

$\Rightarrow D_f=\left\{x \mid x=k,\ k\in \dfrac{4w-5}{-2w+3}\right\}$