

$$2x^2 + y^2 - 4x + 4y + k = 2x^2 - 4x + y^2 + 4y + k = 0 \Rightarrow 2(x^2 - 2x) + y^2 + 4y + k = 0$$

مربع کامل

$$\begin{cases} 2(x^2 - 2x) \rightarrow 2((x-1)^2 - 1) = 2(x-1)^2 - 2 \\ (y+2)^2 - 4 \end{cases} \Rightarrow 2(x-1)^2 + (y+2)^2 - 6 + k = 0$$

تبع نقطه (۱, ۳) فقط به ازای $K=11$ معادله مثبت

$$\Rightarrow 2(x-1)^2 + (y+2)^2 = 11 - k \Rightarrow 11 - k > 0 \Rightarrow K < 11$$

(۱, ۵)

$$f(x) = \begin{cases} x^2 + 5x & ; x \geq a \\ 2x + a + 2 & ; x \leq a \end{cases} \xrightarrow{x=a} \begin{cases} a^2 + 5a \\ 2(a) + a + 2 \end{cases} \Rightarrow a^2 + 5a = 2a + 2 \Rightarrow a^2 + 3a - 2 = 0 \Rightarrow (a+2)(a-1) = 0$$

$$\Rightarrow a = \begin{cases} +1 \\ -2 \end{cases} \xrightarrow{a > 0} a = 1 \Rightarrow f(1) = 2(1) + 1 + 2 = 5 \checkmark$$

۲ (۲)

$$f(x) = \begin{cases} 2x^2 - b & ; x \geq 1 \\ 2x + a & ; x \leq -2 \end{cases} \xrightarrow{x=-2} 2x^2 - b = 2x + a \Rightarrow 1 - b = -4 + a$$

$$\Rightarrow a + b = 1 + 4 = 5 \checkmark$$

۳ (۲)

$$y = \sqrt{4x-a} + \sqrt{b-2x} \Rightarrow \begin{cases} 4x-a \geq 0 \rightarrow x \geq \frac{a}{4} \text{ (I)} \\ b-2x \geq 0 \rightarrow x \leq \frac{b}{2} \text{ (II)} \end{cases} \Rightarrow D_y = \left\{ x \mid \frac{a}{4} \leq x \leq \frac{b}{2} \right\}$$

II I

$$\Rightarrow \frac{a}{4} = \frac{b}{2} = 2 \Rightarrow \begin{cases} a = 8 \\ b = 4 \end{cases} \Rightarrow \frac{b}{a} = \frac{4}{8} = \frac{1}{2} \checkmark$$

۴ (۲)

الف) $\begin{cases} \frac{x+1}{(x-3)^2} \geq 0 \xrightarrow{x+1 \geq 0} x \geq -1 \\ \frac{4-x}{x^2+2x+4} \geq 0 \end{cases} \Rightarrow \{x \mid x \geq -1, x \neq 3\} \xrightarrow{\text{I}} D_f = [-1, 4] - \{3\}$

ب) $\begin{cases} [x] - 2 \geq 0 \Rightarrow [x] \geq 2 \Rightarrow x \geq 4 \\ 2 - [x] \geq 0 \Rightarrow 2 \geq [x] \Rightarrow x < 3 \end{cases} \xrightarrow{\text{اشتراک}} D_f = \emptyset \checkmark$

۵ (۲)

الف) $\frac{x^2 - x - 4}{x^2 - 13x^2 + 4} \geq 0 \Rightarrow \frac{(x-3)(x+1)}{(x^2-9)(x^2-4)} = \frac{(x-3)(x+1)}{(x-3)(x+3)(x-2)(x+2)} \geq 0 \Rightarrow \frac{-}{+} \frac{-}{-} \frac{+}{-} \frac{+}{+}$
 $\Rightarrow D_f = (-\infty, -3) \cup (-2, 2) \cup (3, +\infty)$

ب) $\frac{x^2 - 2x^2 - 8x + 4}{x^2 + 2x^2 - 8x - 4} \geq 0 \Rightarrow \frac{(x-1)(x^2 - x - 4)}{(x+1)(x^2 + x - 4)} \geq 0 \Rightarrow \frac{-}{+} \frac{-}{-} \frac{+}{-} \frac{+}{+}$
 $\Rightarrow D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [4, +\infty)$

الف) $y = \sqrt{[x] - 2} \Rightarrow [x] - 2 \geq 0 \Rightarrow [x] \geq 2 \Rightarrow -x \geq 2 \Rightarrow x \leq -2 \Rightarrow D_f = (-\infty, -2]$

ب) $y = \frac{\sin x + 1}{[x]^2 - 2[x] + 1} \Rightarrow [x]^2 - 2[x] + 1 \neq 0 \Rightarrow ([x] - 1)^2 \neq 0 \Rightarrow [x] - 1 \neq 0 \Rightarrow [x] \neq 1$
 $\Rightarrow \begin{cases} [x] \neq 1 \Rightarrow x \neq [1, 2) \\ [x] \neq -1 \Rightarrow x \neq [-1, 0) \end{cases} \Rightarrow D_f = \mathbb{R} - \{[-1, 0) \cup [1, 2)\}$

الف) $y = \frac{\cot x + 1}{\tan x + 1} \Rightarrow \begin{cases} \sin x \neq 0, \cos x \neq 0 \\ \tan x \neq -1 \end{cases} \Rightarrow D_f = \mathbb{R} - \left\{ \frac{k\pi}{2}, \frac{(2k+1)\pi}{4} \right\}$

ب) $y = \sqrt{1 - \sin^2 x} \Rightarrow (1 - \sin x)(1 + \sin x) \geq 0 \Rightarrow -\frac{1}{-1} \leq \sin x \leq \frac{1}{1} \Rightarrow -1 \leq \sin x \leq 1$
 $\Rightarrow D_f = [k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2}]$

الف) $\sqrt{1 - \log_{\frac{1}{2}} x - 1} \Rightarrow 1 - \log_{\frac{1}{2}} x \geq 0 \Rightarrow \log_{\frac{1}{2}} x \leq 1 \Rightarrow x - 1 \leq \frac{1}{2} \Rightarrow x \leq \frac{3}{2}$
 $x - 1 > 0 \Rightarrow x > 1 \Rightarrow D_f = [1, \frac{3}{2}]$

ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_{\frac{1}{2}} x} \Rightarrow \begin{cases} x(x-1) \geq 0 \\ \log_{\frac{1}{2}} x \neq 1 \end{cases} \Rightarrow \frac{+}{+} \frac{-}{-} \frac{+}{+} \Rightarrow x^2 - x \geq 0 \Rightarrow x(x-1) \geq 0 \Rightarrow x \leq 0 \text{ or } x \geq 1$
 $\log_{\frac{1}{2}} x \neq 1 \Rightarrow x \neq \frac{1}{2} \Rightarrow D_f = (-\infty, 0) \cup (1, +\infty) - \{\frac{1}{2}\}$

الف) $y = \sqrt{x^2 - x^2 - 1} \Rightarrow x^2 - x^2 \geq 1 \Rightarrow 0 \geq 1 \Rightarrow \text{No solution}$
 $\Rightarrow D_f = \emptyset$

ب) $\left(\frac{x+8}{x+5}\right)! \Rightarrow \frac{x+8}{x+5} \in \mathbb{N} \Rightarrow x \in \frac{(n-8)(n-5)}{n-2}$
 $\Rightarrow D_f = \left\{ x \mid x = \frac{(n-8)(n-5)}{n-2}, n \in \mathbb{N} \right\}$

لكنه ليس نون!