

پارامترهای مجهول - بار دوم پس از A - تالیف 11

$$S = \alpha + \beta \stackrel{\beta = 2\alpha}{=} (\alpha = \frac{a}{3} \Rightarrow) a = 12\alpha$$

$$P = 2\beta \stackrel{\beta = 2\alpha}{=} 4\alpha \Rightarrow \alpha = \frac{P}{4} \Rightarrow \alpha = \pm \frac{P}{4} \Rightarrow a = \pm 12$$

$$|12 - (-12)| = 24$$

$$\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{ab}} = \frac{\sqrt{(\sqrt{a} + \sqrt{b})^2}}{\sqrt{ab}} = \sqrt{\frac{a+b+2\sqrt{ab}}{ab}} = \sqrt{\frac{S+2P}{P}} \stackrel{S=\frac{m+1}{2}}{=} \sqrt{\frac{\frac{m+1}{2}+2P}{P}} \stackrel{P=\frac{1}{3}}{=} \sqrt{\frac{\frac{m+1}{2}+\frac{2}{3}}{\frac{1}{3}}} = \sqrt{\frac{m+1}{1}} = \sqrt{m+1} = 8 \Rightarrow m+1=64 \Rightarrow m=63$$

موضوعی $\alpha > \beta$

$$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$\Rightarrow 1 - (\alpha - \beta)^2 = -1 \Rightarrow (\alpha - \beta)^2 = 2 \Rightarrow |\alpha - \beta| = \sqrt{2} \Rightarrow \alpha - \beta = \sqrt{2} \quad (I)$$

$$\alpha + \beta = 1$$

$$\alpha - \beta = \sqrt{2}$$

$$\Rightarrow \alpha = \frac{1+\sqrt{2}}{2}, \beta = \frac{1-\sqrt{2}}{2}$$

معادله β, α ریشه های معادله

$$\begin{aligned} x^2 - 2x + 1 &= 0 \\ x = 1 \end{aligned}$$

$$\begin{aligned} 3\alpha^2 + 2\beta^2 &= (2\alpha^2 + 2\beta^2) + (\alpha^2) = 2(\alpha^2 + \beta^2) + \alpha^2 \\ &= 2(S - 2P) + \alpha^2 = 2(1 - 2P) + \alpha^2 = 2(1 - 2 \cdot \frac{1}{2}) + \alpha^2 = 2(0) + \alpha^2 = \alpha^2 \end{aligned}$$

d - با توجه به فرم معادله $(x^2 - vx - d = 0)$ اگر α ریشه های معادله باشد، $a = -v$ نیز ریشه های معادله خواهد بود. پس جمع ریشه های معادله (S) صفر خواهد بود $\Rightarrow S = 0$

$$2P^2 - 3SP + 2S \stackrel{S=0}{=} 2P^2$$

$$x^2 - vx - d = 0 \Rightarrow x = \frac{v \pm \sqrt{v^2 + 4d}}{2} \Rightarrow x = \frac{v \pm \sqrt{69}}{2}$$

$$\begin{aligned} \alpha &= \frac{v + \sqrt{69}}{2} \\ \beta &= \frac{v - \sqrt{69}}{2} \end{aligned} \Rightarrow \alpha^2 + \beta^2 = \frac{v^2 + 69}{2} + \frac{v^2 - 69}{2} = v^2$$

دو ریشه مستقیم α و β را مساوی با $\frac{v \pm \sqrt{69}}{2}$ گذاشت اما خواستیم نشان دهیم که واقعاً دو ریشه هستند (قریبی یکدیگرند)

$$\begin{aligned} \alpha &= \frac{v + \sqrt{69}}{2} \\ \beta &= \frac{v - \sqrt{69}}{2} \end{aligned} \Rightarrow \alpha^2 + \beta^2 = \frac{v^2 + 69}{2} + \frac{v^2 - 69}{2} = v^2$$

$$\begin{aligned} S_1 &= \alpha + \beta = v \\ P_1 &= \alpha\beta = \frac{v^2 - 69}{4} \end{aligned} \Rightarrow \alpha^2 + \beta^2 = v^2 \Rightarrow v^2 = \frac{v^2 - 69}{4} \Rightarrow v^2 = -69 \Rightarrow v = \pm \sqrt{-69}$$

پارا ۱۹ - جابجایی - یازدهم - A - تالیف

نوع ۱: $a=2$

$$ax^2 - ax - b = 0 \implies 20x^2 - 10x - b = 0 \xrightarrow{x=\alpha} 20\alpha^2 - 20\alpha - b = 0 \implies 20\alpha^2 = 20\alpha + b \quad (I)$$

$$20\beta^2 + 20\alpha^2 - 20\beta = 14 \quad (I) \text{ و } (II) \implies 20\beta^2 + 20\alpha^2 - 20\beta = 14 \implies 20\alpha^2 + 20\beta^2 - 20\alpha - 20\beta = 14$$

$$S = \alpha + \beta = -\frac{-20}{20} = 1 \quad (III)$$

$$(III) \implies 20 + 20b = 14 \implies b = -1$$

$$\implies ax^2 - ax - b = 20x^2 - 20x + 1$$

$$\implies |\alpha - \beta| = \frac{\sqrt{5}}{|a|} = \frac{\sqrt{400 - 10}}{20} = \frac{\sqrt{390}}{20} = \frac{1\sqrt{39}}{20} = \frac{2\sqrt{39}}{40}$$

معادله I: $x^2 - (a+1)x + a = 0 \xrightarrow{\text{جمع ضرایب} = 0} \begin{cases} x_1 = 1 \\ x_2 = a \end{cases} \xrightarrow{\text{دو عدد در معادله طبیعی}} \boxed{a=3} \implies P_1 = a = 3 \quad (IV)$

معادله II: $x^2 - (2a+1)x + b = 0 \xrightarrow{(I)} x^2 - 10x + b = 0 \implies (x-\alpha)(x-\beta) = 0 \implies x^2 - (\alpha+\beta)x + \alpha\beta = 0 \quad (III)$

$$\implies \begin{cases} \alpha + \beta = 10 \\ \alpha\beta = b \end{cases} \xrightarrow{\text{معادله درجه دوم}} \begin{cases} 2\alpha + 2 = 10 \implies \alpha = 4 \\ \beta = 6 \end{cases} \implies b = (4)(6) = 24 \implies P_2 = b = 24 \quad (V)$$

$$(IV), (V) \implies P_2 - P_1 = 24 - 3 = 21$$

۹ - $0 \geq 0 \implies b^2 - 4ac \geq 0 \implies 1 + 4 + m^2 \geq 0 \implies m^2 + 5 \geq 0 \checkmark$ معادله برقرار است

کمترین مقدار $\alpha^2 + \beta^2$ در انزای کمترین مقدار $m^2 = 0$ یعنی $m^2 = 0$ است

$$\min(\alpha^2 + \beta^2) = 0 + 3 = 3$$

۱۰ - $A(3, y)$ و $B(-5, y)$ نقطه A و B روی یک خط عمود بر محور y دارند $\implies x_s = \frac{x_A + x_B}{2} = \frac{3 - 5}{2} = -1 \implies \text{ext}(-1, 0)$

$$\implies y = a(x+1)^2 + 1 \implies y = ax^2 + 2ax + a + 1$$

$$\alpha^2 + \beta^2 = S^2 - 2P = 4 - 2\left(\frac{a+1}{2}\right) = 2 - (a+1) = 1 - a \implies 2\left(\frac{a+1}{2}\right) = -1 \implies a+1 = -1 \implies a = -2$$

$$\implies y = -\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2} \implies y = \frac{1}{2}$$