

$$\begin{aligned} & \text{برشماره} \rightarrow x_1, x_2 \rightarrow x_1 = 2x_2 \Rightarrow x_1 x_2 = 2x_2^2 = \frac{c}{a} = \frac{4}{3} \\ \Rightarrow x_2^2 &= \frac{4}{9} \Rightarrow x_2 = \begin{cases} +\frac{2}{3} \\ -\frac{2}{3} \end{cases} \Rightarrow x_1 = 2 \Rightarrow x_1 + x_2 = \frac{1}{3} \\ \Rightarrow \frac{a}{3} &= \begin{cases} \frac{1}{3} \\ -\frac{1}{3} \end{cases} \Rightarrow \boxed{a = \pm 1} \Rightarrow |1 - (-1)| = \boxed{19} \checkmark \end{aligned}$$

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$$\begin{aligned} \alpha, \beta &\Rightarrow \sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 2 \\ \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} &= 2 \Rightarrow \frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 2 \end{aligned}$$

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$$\begin{aligned} \Rightarrow \frac{m+1}{24} + 2\sqrt{49} &= 2 \Rightarrow m = -1 \Rightarrow mx^2 + 2x + 1 = 0 \Rightarrow -x^2 + 2x + 1 = 0 \\ \Rightarrow p &= \boxed{-2} \checkmark \end{aligned}$$

$$\alpha + \beta = 1 \Rightarrow \alpha\beta = -2 \Rightarrow x^2 - x - 2$$

$$4x^2 + kx^2 - 9x - 2 \mid \frac{x^2 - x - 2}{4x + 1} \Rightarrow (x^2 - x - 2)(4x + 1) = 4x^2 - 3x - 2$$

$$\boxed{k = -3} \checkmark$$

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$$\begin{aligned} \alpha\beta = a, \alpha + \beta = -4 &\Rightarrow \beta = -4 - \alpha \Rightarrow 3\alpha^2 + 2(-4 - \alpha)^2 = 3\alpha^2 + 2(16 - 8\alpha + \alpha^2) \\ 0\alpha^2 + 16\alpha + 32 &= 12\sqrt{2} + 10 \Rightarrow 2\alpha^2 + 16\alpha - 12 = 12\sqrt{2} \end{aligned}$$

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$$\begin{aligned} \Rightarrow \alpha\beta = a &= (-2\sqrt{2} - 3)(-2 + 2\sqrt{2}) = 4\sqrt{2} - 1 + 9 - 4\sqrt{2} = 8 \\ \beta &= -2 + 2\sqrt{2} \end{aligned}$$

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$$x^2 = t \Rightarrow t^2 - vt - 2 = 0 \Rightarrow x^2 = \begin{cases} \frac{v+\sqrt{v^2+8}}{2} \\ \frac{v-\sqrt{v^2+8}}{2} \end{cases} \text{ دو به دو}$$

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$$\begin{aligned} \Rightarrow x &= \pm \sqrt{\frac{v+\sqrt{v^2+8}}{2}} \\ 2p^2 - 2sp + 2s &= 2\left(-\frac{v+\sqrt{v^2+8}}{2}\right)^2 - 0 + 0 = \boxed{29 + 4\sqrt{49}} \checkmark \end{aligned}$$

$$\begin{aligned} 0 < 0, 0 < 1 &\Rightarrow \frac{a}{r} = -\frac{a}{ra} + 1 = -\frac{1}{r} + 1 \geq \frac{1}{r} \Rightarrow a \geq 1 \\ \Rightarrow ra^r + a^r - 1 &= 0 \rightarrow ra^r + a^r - 1 \geq 0 \rightarrow a \geq \frac{1}{r} \\ \Rightarrow ra^r - a + b &= 0 \xrightarrow{a \geq \frac{1}{r}} 1 - r + b \geq 0 \Rightarrow b \geq r - 1 \Rightarrow \left[\frac{ab}{r} \right] \geq \left[-\frac{r}{r} \right] = -1 \end{aligned}$$

$$\begin{aligned} \alpha + \beta &\geq \frac{-(-a)}{a} = 1, \alpha\beta \geq -\frac{b}{a} \quad \alpha a^r + a\alpha - b = 0 \Rightarrow \alpha^r = \alpha + \frac{b}{a} \\ \Rightarrow \alpha(\beta + \frac{b}{a}) + r_0(\alpha + \frac{b}{a}) - r_0\beta &\geq r_0\beta + r_0\alpha + 4 \cdot \frac{b}{a} = r_0(\alpha + \beta) + 4 \cdot \frac{b}{a} \\ = r_0 + 4 \cdot \frac{b}{a} &\geq 1 \vee \Rightarrow 4 \cdot \frac{b}{a} \geq -r \Rightarrow \frac{b}{a} \geq -\frac{1}{r_0} \Rightarrow \alpha\beta \geq -\frac{1}{r_0} \\ (\alpha - \beta)^r &\geq (\alpha + \beta)^r - r\alpha\beta \geq 1 - r \times \frac{1}{r_0} = \frac{r}{r_0} \Rightarrow |\alpha - \beta| \geq \frac{r}{\sqrt{r_0}} \end{aligned}$$

$$\begin{aligned} a+1 &\geq r + (r+1) \geq r+1 \\ a &\geq r(r+1) \geq r^r + r \end{aligned} \rightarrow r^r - r \geq -1 \geq r^r \geq 1 \Rightarrow r \geq -1$$

$$\Rightarrow a \geq r \rightarrow r^r - 1 \cdot r + b \geq 0$$

$$\begin{aligned} b &\geq r(r+1) \geq r^r + r \\ 1 &\geq r + (r+1) \geq r^r + r \end{aligned} \left\{ \begin{aligned} b - 1 &\geq r^r + r - r \\ \frac{r}{r+1} &\geq \frac{r}{r+1} \end{aligned} \right. \Rightarrow b = r^r$$

$$\Rightarrow r^r - r = (r) \checkmark$$

$$\begin{aligned} \alpha + \beta &\geq 1, \alpha\beta \geq -1 - m^r \quad \alpha^r + \beta^r \geq (\alpha + \beta)^r - r\alpha\beta \geq 1 - r(-1 - m^r) = \\ &\geq r m^r + r \Rightarrow \min \geq r x_0 + r \geq (r) \checkmark \end{aligned}$$

$$y = a(x - (-1)^r + 1) \geq ax^r + rax + 1 + a$$

$$s = -r \quad p = \frac{1+a}{a} \quad \alpha^r + \beta^r \geq (\alpha + \beta)^r - r\alpha\beta \geq s^r - rp \geq r - \frac{r+a}{a}$$

$$\Rightarrow a \geq -\frac{r}{r} \Rightarrow y = -\frac{r}{r} x^r = -\frac{r}{r} x + \frac{1}{r} \Rightarrow \text{min } y = \left(\frac{1}{r} \right) \checkmark$$