

$$y = -x^2 + bx + 3 \quad \text{max} = 4$$

$$\rightarrow \text{max}_{\text{y}} = -\frac{\Delta}{4a} = 4 \Rightarrow \frac{b^2 + 12}{4} = 4$$

$$\rightarrow b^2 + 12 = 16 \rightarrow b^2 = 4 \rightarrow b = \pm 2$$

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$$\text{الف) } f(x) = x^2 + 2x + 3 \rightarrow \Delta = 4 - 12 < 0$$

پس تابع صفر ندارد.

$$\text{ب) } f(x) = (4 - x^2)^2 - 2(4 - x^2) - 15$$

$$\rightarrow \text{if } 4 - x^2 = t \rightarrow t^2 - 2t - 15 = 0 \rightarrow (t - 5)(t + 3) = 0$$

$$\begin{cases} 4 - x^2 = 5 \rightarrow x^2 = -1 \quad \times \\ 4 - x^2 = -3 \rightarrow x^2 = 7 \end{cases}$$

$$\rightarrow x = \pm \sqrt{7}$$

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$$x^2 - 14x + m = 0$$

$$\alpha = \beta + 2$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-14}{1} = 14 \quad \alpha = \beta + 2 \Rightarrow 2\beta + 2 = 14 \rightarrow \beta = 6, \alpha = 8$$

$$\rightarrow \alpha \cdot \beta = \frac{c}{a} = 1 \cdot m = 14 \rightarrow m = 14$$

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$$3x^2 - 4x + m - 1 = 0$$

$$2\alpha - \beta = 4 \rightarrow \alpha = \frac{4 + \beta}{2}$$

$$\rightarrow \alpha + \beta = \frac{4 + \beta}{2} + \beta = \frac{4 + 3\beta}{2} = -\frac{b}{a} = 2 \rightarrow \beta = 0, \alpha = 2$$

$$\rightarrow \alpha \beta = 2(0) = 0 = \frac{m - 1}{3} \rightarrow m - 1 = 0 \rightarrow m = 1$$

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$$-mx^2 + 4x + m^2 - 2 = 0$$

$$\alpha = \frac{1}{\beta}$$

$$\alpha \cdot \beta = \beta \times \frac{1}{\beta} = 1 = \frac{c}{a} = \frac{m^2 - 2}{-m} \rightarrow -m = m^2 - 2$$

$$\rightarrow m^2 + m - 2 = 0 \rightarrow (m + 2)(m - 1) = 0 \begin{cases} m = 1 \rightarrow \Delta > 0 \checkmark \\ m = -2 \rightarrow \Delta = 0 \text{ فقط یک ریشه دارد. } \times \end{cases}$$

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$$x^2 - mx + 3 = 0$$

$$-\alpha\beta^2 = 3$$

$$\alpha\beta = \frac{c}{a} = 3 \rightarrow \alpha\beta^2 = \alpha\beta \times \beta = 3\beta = 3 \rightarrow \beta = \frac{3}{3} = 1$$

$$\xrightarrow{\alpha\beta=3} \alpha = \frac{3}{\beta} \rightarrow \alpha + \beta = \frac{3}{\beta} + \beta = \frac{3+14}{12} = \frac{17}{12} = -\frac{b}{a}$$

$$\rightarrow -\frac{-m}{1} = \frac{17}{12} \rightarrow m = \frac{17}{12}$$

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$$x^2 - 5x + m = 0$$

$$\alpha = 3\beta$$

$$\alpha + \beta = -\frac{b}{a} = 5 \xrightarrow{\alpha=3\beta} \beta + 3\beta = 5 \rightarrow \beta = 1, \alpha = 3$$

$$\rightarrow \alpha\beta = \frac{c}{a} = m \rightarrow m = 3(1) = 3$$

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$$x^2 - 4x + 2 = 0$$

$$\alpha^2 + \frac{1}{\alpha^2} = ?$$

$$S = 4$$

$$P = 2$$

$$\alpha\beta = 2 \rightarrow \alpha = \frac{2}{\beta} \xrightarrow{\alpha^2} \alpha^2 = \frac{4}{\beta^2} \rightarrow \beta^2 = \frac{4}{\alpha^2}$$

$$\rightarrow \alpha^2 + \frac{1}{\alpha^2} = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4^2 - 2(2) = 16 - 4 = 12$$

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$$h(t) = -10t^2 + 40t$$

$$\max h = -\frac{a}{4a} = -\frac{200 + 0}{-40} = \frac{200}{40} = 50, 40 \text{ m}$$

$$\text{وقتی توپ به زمین می خورد} = \text{ریشه بزرگتر} \rightarrow -10t^2 + 40t = 0 \rightarrow 10t(-t + 4) = 0$$

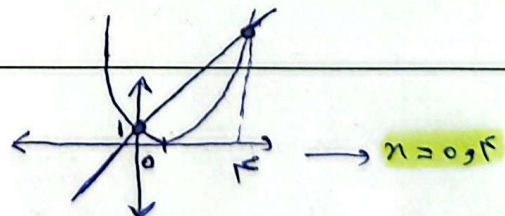
$$t = 0 \text{ هنگام شروع}$$

$$t = 4 \text{ هنگام برخورد}$$

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$$\text{الف) } (n-1)^2 = 2n+1$$

ریشه 1



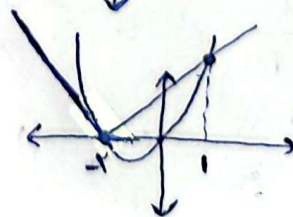
$$n = 0, 3$$

$$\text{ب) } n^2 + 2n = |n+2|$$

$$n(n+2)$$

$$0 - 2$$

$$-2$$



$$n = -2, 1$$

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