ریاضی تکمیلی یازدهم A

-۱

$y=(a+1)x^2+x+3$

$-\frac{b}{2a}=2$ $\qquad \frac{-1}{2(a-1)}=2 \Rightarrow -4a+4=1 \Rightarrow -4a=-3 \Rightarrow a=\frac{3}{4}$

$y=-\frac{1}{4}x^2+x+3 \xrightarrow{\times -4} x^2-4x-12=0 \Rightarrow \begin{matrix} x_1=-2 \\ x_2=6 \end{matrix}$

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$f(x)=mx^2-5x+m$

$f(x)$		x_1		x_2	
x	$-$	|	$+$	|	$-$

$\Delta>0 \Rightarrow b^2-4ac>0 \Rightarrow 25-4m^2>0 \Rightarrow m^2<\frac{25}{4} \Rightarrow -\frac{5}{2}<m<\frac{5}{2}$

$a<0 \Rightarrow m<0 \longrightarrow$

$-\frac{5}{2}<m<0$

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$x_1=\frac{3+\sqrt{5}}{3}\ ,\ x_2=\frac{3-\sqrt{5}}{3}$

$\begin{cases} x_1-\left(\frac{3+\sqrt{5}}{3}\right)=0 \\ x_2-\left(\frac{3-\sqrt{5}}{3}\right)=0 \end{cases}$

$\left(x-\left(\frac{3+\sqrt{5}}{3}\right)\right)\left(x-\left(\frac{3-\sqrt{5}}{3}\right)\right)=x^2+x\left(\frac{-3+\sqrt{5}-3-\sqrt{5}}{3}\right)+(9-5)=x^2+x\left(\frac{-6}{3}\right)+\frac{4}{3}=x^2-2x+\frac{4}{3}$

$\longrightarrow 3x^2-6x+4$

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$2x^2-8x+m+2 \qquad \alpha<\beta \qquad 3\alpha^2+\beta^2=12$

$3\alpha^2+\beta^2=12 \Rightarrow (\alpha^2+\beta^2)+2\alpha^2=12 \Rightarrow (S^2-2P)+2\alpha^2=12$

$\left(-\left(\frac{-8}{2}\right)\right)^2-2\left(\frac{m+2}{2}\right)+2\alpha^2=12 \Rightarrow 16-m-2+2\alpha^2=12 \xrightarrow{\alpha<\beta} \alpha=\frac{-b-\sqrt{\Delta}}{2a}=\frac{8-\sqrt{64-8(m+2)}}{4}$

$=\frac{4-\sqrt{16-2m-4}}{2} \Rightarrow 2\alpha^2=2\left(\frac{16-2m-4-8\sqrt{16-2m-4}}{4}\right)=6-m-4\sqrt{16-2m-4}$

$\Rightarrow 16-m-2+6-m-4\sqrt{16-2m-4}=12$

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$S(2,9) \quad c=5 \qquad \rightarrow \left(\frac{h}{k}\right): a(x-h)^2+k \qquad a(x-2)^2+9=a(x^2-4x+4)+9=ax^2-4ax+\underbrace{4a+9}_{5}$

$\Rightarrow 4a+9=5 \Rightarrow a=-1 \qquad -(x-2)^2+9=-x^2+4x-4+9=-x^2+4x+5$

$-x^2+4x+5>0 \qquad x^2-4x-5<0$

$(x-5)(x+1)<0$

$f(x)$		-1		5	
x	$+$	|	$-$	|	$+$

$(-1,5)$

$\alpha x^2 - 7x + 9\beta = 0 \qquad \frac{1}{\alpha}, \frac{1}{\beta}$

$P = \alpha\beta = \frac{9\beta}{\alpha} \Rightarrow \alpha^2\beta = 9\beta \qquad \beta(\alpha^2 - 9) = 0 \begin{cases} \beta = 0 \to \frac{1}{\beta} = 0 \text{ غ ق ق} \\ \alpha = \pm 3 \end{cases}$

$\alpha = 3 \to \frac{7}{\alpha} = \alpha + \beta \Rightarrow 7 = \alpha^2 + \alpha\beta \Rightarrow \alpha\beta = -2 \to \beta > 0, \ \alpha < 0$

$\alpha = -3 \to -3 \times \beta = -2 \Rightarrow \beta = \frac{2}{3}$

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$x^2 + mx - 2m = 0 \qquad S = \frac{-b}{a} = -m$

$\alpha^2 - m\beta = 8$

$\alpha^2 + (\alpha+\beta)\beta = 8 \Rightarrow \alpha^2 + \beta^2 + \alpha\beta = 8 \Rightarrow S^2 - 2P + P = 8 \Rightarrow S^2 - P = 8$

$(-m)^2 - (-2m) = 8 \qquad m^2 + 2m - 8 = 0 \Rightarrow (m+4)(m-2) = 0 \Rightarrow m = 2, \ m = -4$

$\Delta > 0 \Rightarrow m = -4 \Rightarrow (-4)^2 - 32 = 16 - 32 = -16$ غ ق ق

$m = 2 \Rightarrow 4 + 16 = 20 > 0$ ✓ $\qquad S = \frac{-b}{a} = -m = -2$

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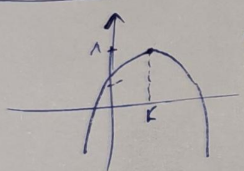

$f(x) = mx^2 + 4x + \frac{m}{2} + 7$

$y_{ext} = \frac{-\Delta}{4a} = \frac{16 - 4m(\frac{m}{2} + 7)}{4m} = 8 \Rightarrow \frac{m^2}{2} - 8m + 7m - 4 = 0$

$\Rightarrow \underbrace{m^2 - 2m - 8}_{(m-4)(m+2)} = 0 \Rightarrow m = 4, \ m = -2 \qquad a < 0 \Rightarrow m < 0 \Rightarrow m = -2$

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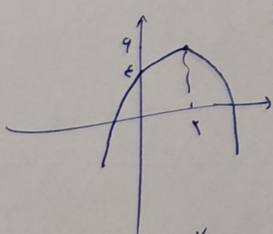

$y = a(x-h)^2 + k$

$y = a(x-2)^2 + 9 \overset{x=0}{\Rightarrow} 4 = a(0-2)^2 + 9 \Rightarrow a = -\frac{1}{2}$

$y = -\frac{1}{2}(x-2)^2 + 9 = -\frac{x^2}{2} + 2x + 4 = 0 \Rightarrow x^2 - 4x - 8 = 0$

$S = -\frac{(-4)}{1} = +4$

$P = \frac{-8}{1} = -8$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{S}{P} = \frac{4}{-8} = \boxed{-\frac{1}{2}}$

$x^۲ - (۴a+۴)x + (۳a^۲ + ۶a + ۳) = ۰ \rightarrow x = ۲$

$x = ۲ \Rightarrow ۴ - ۸a - ۸ + ۳a^۲ + ۶a + ۳ = ۰ \Rightarrow ۳a^۲ - ۲a - ۱ = ۰ \xRightarrow{a+b+c=۰} x_۱ = ۱ ,\quad x_۲ = \frac{c}{a} = -\frac{۱}{۳}$

$\Delta > ۰ \begin{cases} a = ۱ \Rightarrow x^۲ - ۵x + ۱۲ = ۰ \Rightarrow \Delta = ۲۵ - ۴۸ = -۲۳ \text{ غق} \\ a = -\frac{۱}{۳} \Rightarrow x^۲ - \frac{۸}{۳}x + \frac{۲}{۳} = ۰ \xrightarrow{\Delta > ۰} S = \alpha + \beta = \frac{۸}{۳} \end{cases}$

$\alpha + ۲ = \frac{۸}{۳} \Rightarrow \alpha = \frac{۲}{۳}$