

$y = (a-1)x^2 + x + 3$
 $\rightarrow -1 = f(a-1) = fa - f = -1 \rightarrow fa = 3 \rightarrow a = \frac{3}{f}$
 $\rightarrow \frac{-b}{2a} = 2 \rightarrow -b = 4a$
 $\Rightarrow y = (\frac{3}{f}-1)x^2 + x + 3 = -\frac{1}{f}x^2 + x + 3 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1+3}}{-\frac{1}{f}} = \frac{-1 \pm 2}{-\frac{1}{f}}$
 $\Rightarrow x = -3/-\frac{1}{f} \rightarrow x = 4$ ✓
 $\Rightarrow x = 1/-\frac{1}{f} \rightarrow x = -f$ ✓

$f(x) = mx^2 - 5x + m$
 $\Delta > 0 \rightarrow 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \rightarrow m^2 < \frac{25}{4}$
 $\Rightarrow -\frac{5}{2} < m < \frac{5}{2}$
 $m > 0$
 $\Rightarrow m \in (-\frac{5}{2}, 0)$ ✓

$\alpha = \frac{3-\sqrt{5}}{2}$
 $\beta = \frac{3+\sqrt{5}}{2}$
 $\alpha + \beta = 3 = -\frac{b}{a} \rightarrow \frac{3-\sqrt{5}}{2} + \frac{3+\sqrt{5}}{2} = \frac{6}{2} = 3$
 $\alpha \beta = \frac{c}{a} \rightarrow (\frac{3-\sqrt{5}}{2})(\frac{3+\sqrt{5}}{2}) = \frac{9-5}{4} = \frac{4}{4} = 1$
 $\Rightarrow x^2 - 3x + 1 = 0 \rightarrow 4x^2 - 12x + 4 = 0$ ✓

$4x^2 - 12x + m + 2 = 0 \rightarrow 4x^2 - 12x + m + 2 = 0 \rightarrow 4x^2 = 12x - m - 2$
 $\alpha < \beta$
 $4\alpha^2 + 12\alpha + m + 2 = 0$
 $4\beta^2 + 12\beta + m + 2 = 0$
 $\Rightarrow 4\alpha^2 + 12\alpha + m + 2 = 4\beta^2 + 12\beta + m + 2$
 $\Rightarrow 4\alpha^2 - 4\beta^2 + 12\alpha - 12\beta = 0$
 $\Rightarrow 4(\alpha - \beta)(\alpha + \beta) + 12(\alpha - \beta) = 0$
 $\Rightarrow (\alpha - \beta)(4(\alpha + \beta) + 12) = 0$
 $\Rightarrow \alpha + \beta = -3$ ✓

$S = (2, 9)$
 $y = a(x-h)^2 + k$
 $\Rightarrow y = a(x-2)^2 + 9$
 $\Rightarrow C = 5 = fa + 9 \rightarrow a = -1$
 $\Rightarrow x = 1$
 $\Rightarrow x = 5$
 $\Rightarrow -1 < m < 5$
 $m \in (-1, 5)$ ✓

* $\frac{1}{x} + \frac{1}{\beta} = \frac{1}{\gamma} + \frac{1}{\gamma} = \frac{2}{\gamma}$ $\left(A, \frac{1}{\gamma} \right)$

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$\alpha x^2 - \gamma x + 9\beta = 0 \rightarrow \begin{cases} S = \alpha + \beta = \frac{\gamma}{\alpha} \\ P = \alpha\beta = \frac{9\beta}{\alpha} \end{cases} \rightarrow \alpha = \frac{\gamma}{S} \rightarrow P = \frac{9\beta}{\frac{\gamma}{S}} = \frac{9\beta S}{\gamma}$
 $\beta > 0$
 $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{\gamma} \rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\gamma} \rightarrow \frac{S}{\frac{9\beta S}{\gamma}} = \frac{2}{\gamma} \rightarrow \frac{\gamma}{9\beta} = \frac{2}{\gamma} \rightarrow \gamma^2 = 18\beta$
 $\Rightarrow -\gamma\alpha = 9\beta \rightarrow \alpha = -\frac{9\beta}{\gamma}$
 $\Rightarrow P = \alpha\beta = \left(-\frac{9\beta}{\gamma}\right)(\beta) = -\frac{9\beta^2}{\gamma} = P = -\gamma \Rightarrow \beta^2 = \gamma \Rightarrow \beta = \sqrt{\gamma}$
 $\Rightarrow \alpha = -\frac{9\sqrt{\gamma}}{\gamma} = -\frac{9}{\sqrt{\gamma}}$
 $\Rightarrow S = \alpha + \beta = -\frac{9}{\sqrt{\gamma}} + \sqrt{\gamma} = 0 \Rightarrow \sqrt{\gamma} = \frac{9}{\sqrt{\gamma}} \Rightarrow \gamma = 9$
 $\Rightarrow \alpha = -1, \beta = 1 \Rightarrow P = -1$

$\alpha^2 + m\alpha - \gamma m = 0 \rightarrow \alpha^2 + m\alpha - \gamma m = 0 \rightarrow \alpha^2 = -m\alpha + \gamma m$
 $\alpha^2 - m\beta = \lambda \rightarrow \alpha^2 = -m\alpha + \gamma m$
 $\Rightarrow -m\alpha + \gamma m - m\beta = \lambda \rightarrow -m(\alpha + \beta) + \gamma m - \lambda = 0$
 $S = \alpha + \beta = -\frac{\gamma}{m}$
 $\Rightarrow m^2 + \gamma m - \lambda = 0 \rightarrow (m + \gamma)(m - \gamma) = 0 \rightarrow m = -\gamma \rightarrow \Delta > 0$
 $\Rightarrow m = \gamma \rightarrow S = \alpha + \beta = -\frac{\gamma}{m} = -1$

$f(m) = m\alpha^2 + \gamma m + \frac{m}{\gamma} + \gamma$
 $\text{ext of } f = -\frac{\Delta}{f_a} = -\frac{-(14 - f(m)(\frac{m}{\gamma} + \gamma))}{f_m} = \frac{-14 + \gamma m^2 + \gamma \lambda m}{f_m} = \frac{m^2 + \gamma m - \lambda}{\gamma m} = \lambda$
 $\Rightarrow m^2 + \gamma m - \lambda = 14\gamma m \rightarrow m^2 - \gamma m - \lambda = 0 \rightarrow (m - \gamma)(m + \gamma) = 0$
 $m = \gamma \rightarrow f_m^2 + f_m + 9 \rightarrow X$
 $m = -\gamma \rightarrow -\gamma\alpha^2 + \gamma\alpha + 4 \rightarrow \alpha = \frac{-\gamma \pm \sqrt{14 + \gamma}}{-\gamma}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{\gamma} \rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\gamma} \rightarrow \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{2}{\gamma} \rightarrow \frac{-b}{c} = \frac{2}{\gamma} \rightarrow \frac{-b}{c} = -\frac{1}{\gamma}$
 $\alpha x^2 + bx + c \rightarrow \begin{cases} C = F \\ \frac{-b}{\gamma a} = \gamma \rightarrow -b = \gamma a \\ \frac{-\Delta}{f_a} = \frac{b^2 - 4ac}{-4a} = 4 \rightarrow b^2 - 4ac = -4\gamma a \rightarrow b^2 + bc = 4b \end{cases}$
 $\Rightarrow \begin{cases} b + c = 4 \\ c = \gamma \end{cases} \rightarrow b = \gamma \Rightarrow \frac{-b}{c} = \frac{-\gamma}{\gamma} = -1$

$\alpha^2 - (4a + \gamma)\alpha + (3a^2 + 4a + \gamma) = 0$
 $\alpha = 2 \rightarrow 4 - 4a - \gamma + 3a^2 + 4a + \gamma = 0 \rightarrow 3a^2 - \gamma a - 1 = 0 \rightarrow a^2 - \gamma a - \frac{1}{3} = 0$
 $\Rightarrow a = \frac{\gamma \pm \sqrt{\gamma^2 + \frac{4}{3}}}{2}$
 $\Rightarrow a = 1 \rightarrow x^2 - 1x + 1 \rightarrow \checkmark$
 $\Rightarrow a = -\frac{1}{\gamma} \rightarrow x^2 - \frac{1}{\gamma}x + \gamma \rightarrow x = \frac{\frac{1}{\gamma} \pm \sqrt{\frac{1}{\gamma^2} - 4\gamma}}{2}$

$\Rightarrow x^2 - 1x + 1 \rightarrow x = \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm i}{2}$