

$$x^2 + 2x - 8 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 + 32}}{2} = \frac{-2 \pm 6}{2} = -4, 2$$

$$\omega = 2, -4, 2$$

$$L, -2, 2 \leq 2 \Rightarrow \boxed{[-4, 2]}$$

(ب) $(2, -4)$

عدد ۵ را امتحان می کنیم این عدد در معادله کار می کند پس کار می کند
۵ هم بهین عدد ۲ و ۴ است.

$$x^3 - 2 = 12x - 2$$

$$x^3 - 12x + 14 = 0$$

$$\Leftrightarrow x = 2$$

$$\begin{array}{r|l} x^3 - 12x + 14 & x - 2 \\ \hline x^3 - 2x^2 & x^2 + 2x - 8 \\ \hline 2x^2 - 12x + 14 & \\ 2x^2 - 4x & 2x - 8 \\ \hline -8x + 14 & \\ -8x + 16 & \\ \hline -2 & \end{array}$$

$$f_g: y = 3x + k = 3x - 10$$

$$f^{-1}: x = 3y + k \Rightarrow f_y = 4 \Rightarrow 4 = 3(4) + k \Rightarrow k = -10$$

$$f(4) = 11 - 10 = 1$$

$$f(f(x)) = (3x - 10) - 10 = 3x^2 - 20$$

$$f^{-1}: x = \frac{ay}{y-1} \Rightarrow xy - x = ay \Rightarrow \boxed{y = \frac{-x}{x-a}}$$

$$A(2a, a) \Rightarrow 2a = \frac{a(a)}{a-1} \Rightarrow \frac{a^2 - 2a^2 + 2a}{a-1} = 0 \Rightarrow \frac{2a - a^2}{a-1} = \frac{a(2-a)}{a-1} = \frac{a(2-a)}{a-1} = 0 \Rightarrow a = 0, 2$$

$$\Rightarrow \boxed{a=0}, \boxed{a=2}$$

به دلیل صفر شدن منفرجه در تابع f^{-1} است
جواب نه قیاس است و فقط ۲ نقطه می تواند باشد $a=2$

$$f^{-1} = \{(2, 3), (7, 4), (2, 2), (4, 9)\}$$

$$g^{-1} = \{(2, 3), (4, 1), (2, 9)\}$$

$$الف) f \circ f^{-1} = (2, 2), (7, 7), (2, 2), (4, 4)$$

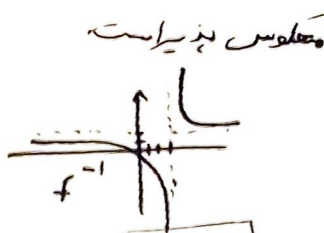
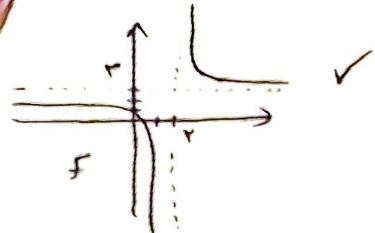
$$ب) f^{-1} \circ f = (3, 3), (4, 4), (7, 7), (9, 9)$$

$$ج) f_g^{-1} = \{(2, 2), (4, 4)\}$$

$$د) g^{-1} \circ f = \{(3, 9), (7, 3), (9, 8)\}$$

$$g^{-1} = \{(1, 2), (3, 4), (2, 4)\} \quad f \circ g^{-1} = \{(1, 4), (3, 8), (2, 2)\}$$

$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{2}), (3, \frac{1}{4}), (2, \frac{1}{2})\} = \boxed{\{(1, \frac{1}{2}), (3, \frac{1}{4})\}}$$



$$f^{-1} \Rightarrow x = \frac{y+1}{y-1} \Rightarrow xy - x = y + 1 \Rightarrow y = \frac{x+1}{x-1}$$

4

ج: $x: 1 \quad 2 \quad D_f = R_{f^{-1}} \quad [a, b] = [1, 2]$

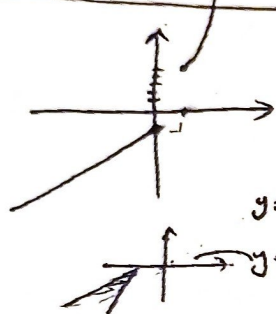
$x: 1$	$x: 2$	$f(x)$
$\frac{1+1}{1-1} = -2$	$\frac{2+1}{2-1} = 3$	$f(x) = \frac{x+1}{x-1}$

$f^{-1} \Rightarrow x = \frac{y+1}{y-1} \Rightarrow y = \frac{x+1}{x-1}$

$1 \leq \frac{x+1}{x-1} \leq 2 \Rightarrow -2 \leq x \leq 3$

$D_{f^{-1}} \rightarrow$

✓



$$y = x + 1 \xrightarrow{f^{-1}} x = y - 1 \Rightarrow y = \sqrt{x - 1}, x \leq 0$$

$$y = \frac{x+1}{x} \xrightarrow{f^{-1}} x = \frac{y}{y-1} \Rightarrow y = \frac{x+1}{x}, x \geq 1$$

$$f^{-1}(x) = \begin{cases} \sqrt{x-1} & ; x \leq 0 \\ \frac{x+1}{x} & ; x \geq 1 \end{cases}$$

✓

$$f(x) = \frac{x^2 + 3x - 1}{x+3} = \frac{-x-1}{x+3}$$

$$f^{-1} \Rightarrow x = \frac{-y-1}{y+3}$$

$$xy + 3x = -y - 1 \Rightarrow y = \frac{-(x+1)}{x+3} = \frac{ax+b}{cx+d}$$

$$f^{-1}(b) = \frac{-(x(-1)+1)}{(-1)+3} = \frac{2}{2} = 1$$

$$\begin{cases} a = -1 \\ b = -1 \\ c = 1 \\ d = 3 \end{cases}$$

$$f^{-1}(b) = \frac{a}{c} = \frac{-1}{1} = -1$$

4

$$f(x) = \frac{x}{x^2+1}$$

$$f^{-1} \Rightarrow x = \frac{y}{y^2+1} \Rightarrow y^2x + x = y \Rightarrow y^2x - y + x = 0$$

$$\Rightarrow x + y - y^2x = 0 \Rightarrow x = \frac{y}{1-y^2}$$

$$y(x+1) = x$$

$$xy^2 - y = -x$$

10

$$f(x) = y \Rightarrow y = \frac{x}{x^2+1} \Rightarrow yx^2 + y = x \Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$y = -y \Rightarrow f^{-1} = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$x = \frac{y}{y^2+1} \Rightarrow y^2x + x - y = 0$$

$$y = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

10

