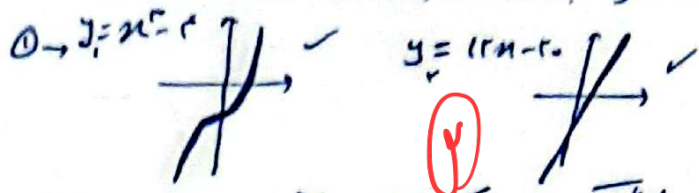


در تمام جملات برای هر یک یک بار در عبارت در بازه‌ی خود یک یک باشد

$$f(x) = \begin{cases} x^2 - 2 & ; x > a \\ 12x - 2 & ; x \leq a \end{cases}$$



استنتاج برد آنها نمی باشد

$$\Rightarrow x^2 - 2 \geq 12x - 2 \Rightarrow x^2 - 12x \geq 0 \Rightarrow x(x - 12) \geq 0 \Rightarrow x \leq 0 \text{ or } x \geq 12$$

$f^{-1}(2) = 2 \Rightarrow f(2) = 2$

$f(x) = 12x - 2 \Rightarrow f(2) = 12 \cdot 2 - 2 = 22 \neq 2$

$f(x) = x^2 - 2 \Rightarrow f(2) = 2^2 - 2 = 2$

الف) $f(v) = ? \Rightarrow f(v) = 22 - 10 = 12$

ب) $f(f(x)) = ? \Rightarrow f(12x - 2) = ? \Rightarrow f(f(x)) = 12(12x - 2) - 2 = 144x - 24 - 2 = 144x - 26$

$f(a) = \frac{a^2}{a-1} = ra \Rightarrow a^2 = ra^2 - ra \Rightarrow a^2 - ra = 0 \Rightarrow a(a-r) = 0$

$\Rightarrow \begin{cases} a = 0 \\ a = r \end{cases}$

$f(x) = \frac{x}{x-a} \Rightarrow f^{-1}(ra) = \frac{ra}{ra-a} = \frac{ra}{a} = r$

$f^{-1}(ra) = a$

$\Rightarrow a = r$

الف) $f \circ f^{-1} = \{(d, d), (v, v), (r, r), (s, s)\}$

ب) $f^{-1} \circ f = \{(r, r), (s, s), (v, v), (d, d)\}$

ج) $f \circ g^{-1} = \{(r, d), (d, s)\}$

د) $g^{-1} \circ f = \{(r, v), (v, s), (s, d)\}$

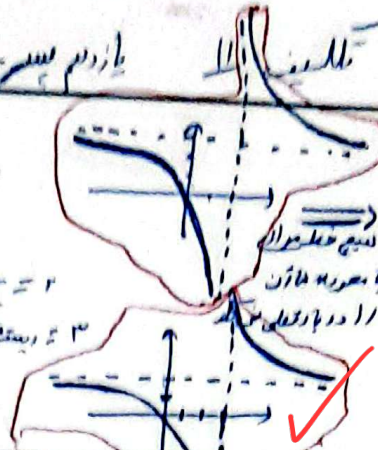
$f \circ g^{-1} = \{(1, 2), (3, 4), (5, 6)\}$

$h = \{(1, \frac{1}{2}), (3, \frac{1}{3}), (5, \frac{1}{5})\}$

$h \circ f^{-1} = \{(1, \frac{1}{2}), (3, \frac{1}{3})\}$

$$y = \frac{x+1}{x-2} \rightarrow \begin{cases} \text{مشتق اول: } \frac{dy}{dx} = \frac{1}{(x-2)^2} \\ \text{مشتق دوم: } \frac{d^2y}{dx^2} = \frac{2}{(x-2)^3} \end{cases}$$

$$f^{-1}(x) = \frac{x+1}{x-2} \quad \begin{cases} \text{مشتق اول: } \frac{dy}{dx} = \frac{1}{(x-2)^2} \\ \text{مشتق دوم: } \frac{d^2y}{dx^2} = \frac{2}{(x-2)^3} \end{cases}$$



تک بر یک است

$$f(x) = |x-1| - |x-3|$$

$$\begin{aligned} & x=1 \rightarrow f(x) = 0 - 2 = -2 \\ & x=3 \rightarrow f(x) = 2 - 0 = 2 \end{aligned}$$

$$f^{-1}(x) = \frac{x+2}{2} = \frac{1}{2}x + 1$$

$$R_{f^{-1}} = D_{f^{-1}} = [-2, 2]$$

$$f(x) = \begin{cases} x^2 + 1 & ; x \geq 1 \\ x-1 & ; x \leq 0 \end{cases}$$

$$\textcircled{1} y_1 = x^2 + 1$$

$$y_2 = x-1$$

$$R_{y_1} = [1, +\infty) \quad R_{y_2} = (-\infty, -1]$$

اشتراک برداشته می باشد

$$f^{-1}(x) = \begin{cases} \sqrt{x-1} & ; x \geq 1 \\ x+1 & ; x \leq -1 \end{cases}$$

$$f(x) = \frac{x^2 + 2x + 1}{x+2} = \frac{(x+1)^2}{x+2}$$

$$\Rightarrow f^{-1}(x) = \frac{-2x-1}{x+2}$$

$$f(x) = \frac{x}{x^2+1} \quad y = \frac{x_1}{x_1^2+1} \quad y = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow x_1 x_2^2 - x_1^2 x_2 + x_1 - x_2 = 0 \Rightarrow (x_1 x_2 - 1)(x_1 - x_2) = 0$$

$$\Rightarrow x_1 = x_2 \quad \text{یا} \quad x_1 x_2 = 1$$

$$f(x) = \frac{x}{x^2+1} \quad f^{-1}(x) = \frac{x}{x^2+1}$$

$$f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$