

$$f(x) = \begin{cases} x^3 - 1 & ; x > a \\ 1(x - 1_0) & ; x \leq a \end{cases} \xrightarrow{x=a} a^3 - 1 \geq 1(a - 1_0) \rightarrow a^3 - 1(a + 1_0) \geq 0$$

$$\Rightarrow (a+1)(a-1)^2 \geq 0 \Rightarrow \frac{-1}{-1+1+1} \Rightarrow a > -1$$

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$$f(x) = 1x + k \text{ و } f^{-1}(1) = f \Rightarrow f(1) = 1 \rightarrow 1 + k = 1 \quad k = -1_0 \Rightarrow f(x) = 1x - 1_0$$

الف) $f(1) = 1 - 1_0 = 0$

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ب) $f(f(x)) = f \circ f(x) = 1(1x - 1_0) - 1_0 = 1x - 1_0$

$$A(1_0, a) \text{ و } f(x) = \frac{ax}{x-1} \Rightarrow A(1_0, a) \rightarrow f(a) = 1_0a$$

$$f(a) = \frac{a^2}{a-1} = 1_0a \rightarrow a^2 = 1_0a^2 - 1_0a \rightarrow a(a-1) = 0 \begin{cases} a=1 \rightarrow X \text{ دامن ندارد} \\ a=0 \rightarrow \checkmark \end{cases}$$

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$$f = \{(3, \omega), (6, \nu), (7, 1), (9, 4)\}, g = \{(3, 1), (1, 4), (9, \omega)\}$$

الف) $f \circ f^{-1} = \{(3, 1), (\omega, \omega), (9, 4), (1, 1)\}$

ب) $f^{-1} \circ f = \{(3, 3), (6, 4), (7, 7), (9, 9)\}$

ج) $f \circ g^{-1} = \{(3, \omega), (\omega, 9)\}$

د) $g^{-1} \circ f = \{(3, 9), (7, 3), (9, 1)\}$

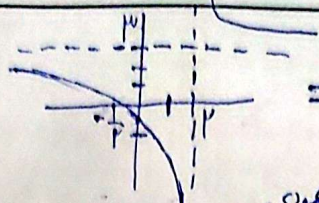
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$$f = \{(2, 1), (1, 1), (9, 0)\} \text{ و } g = \{(2, 1), (1, 3), (4, \omega)\}, h = \{(2, 1), (3, 1), (1, 1), (\omega, 1)\}$$

$$\frac{h}{f \circ g^{-1}} \rightarrow f \circ g^{-1} = \{(2, 1), (3, 1), (\omega, 0)\} \Rightarrow \frac{h}{f \circ g^{-1}} = \{(1, 1), (\omega, 1)\}$$

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$$y = \frac{x^3+1}{x-1} \rightarrow \text{مخالف عددی} = 1 \rightarrow \text{مخالف} = 1^3$$



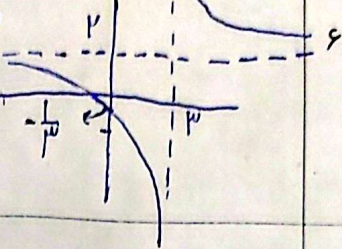
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$$y = \frac{x^3+1}{x-1} \rightarrow x = \frac{y+1}{y-1} \rightarrow xy - 2x = y+1$$

$$\rightarrow y(x-2) = y+1 \rightarrow y = \frac{y+1}{x-2}$$

$$f^{-1}(x) = \frac{x+1}{x-2}$$

مخالف عددی



$$f(x) = |x-1| - |x-2| \rightarrow \begin{cases} 1 & x \geq 2 \\ x-1 & 1 \leq x \leq 2 \\ -1 & x \leq 1 \end{cases} \rightarrow y = x-1 \rightarrow x = \frac{y+1}{1}$$

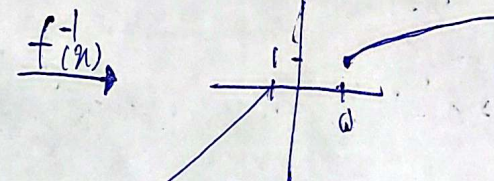
$$\Rightarrow f^{-1}(x) = \frac{x+1}{1}$$

$$f(x) = \begin{cases} x^2+1 & x \geq 1 \\ f(x-1) & x < 1 \end{cases}$$

$$y = x^2+1 \rightarrow x = \sqrt{y-1} \rightarrow f^{-1}(x) = \sqrt{x-1}$$

$$y = f(x-1) \rightarrow x = \frac{y+1}{1} \rightarrow f^{-1}(x) = \frac{x+1}{1}$$

$$f^{-1}(x) = \begin{cases} \frac{x+1}{1} & x < 1 \\ \sqrt{x-1} & x \geq 1 \end{cases}$$



$$f(x) = x^2 - \frac{(x+1)^3}{x+3}, f^{-1}(x) = \frac{ax+b}{cx+d}$$

$$f(x) = \frac{x^2 + 3x^2 - x^3 - 1 - 3x^2 - 3x}{x+3} = \frac{-x^3 - 1}{x+3} \xrightarrow{\text{مکعب}} x = \frac{-y-1}{y+3} \Rightarrow \frac{-y-1}{y+3}$$

$$a = -1, b = -1, c = 1, d = 3$$

$$f^{-1}(-1) = \frac{3(-1)-1}{-1+3} = \boxed{1}$$

$$y = \frac{x}{x^2+1} \rightarrow x = \frac{y}{y^2+1} \rightarrow xy^2+x=y \rightarrow xy^2-y+x=0$$

$$\Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \Rightarrow 1-4x^2 \geq 0 \rightarrow |x| \leq \frac{1}{2} \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}, x \in [-\frac{1}{2}, \frac{1}{2}], x \neq 0$$