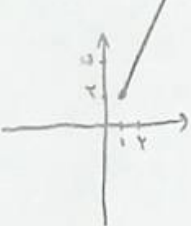
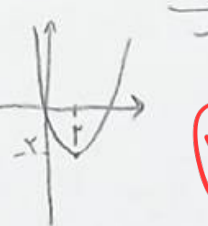
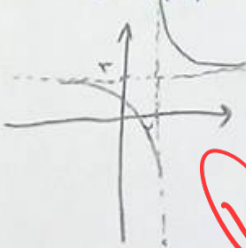
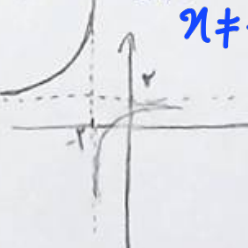
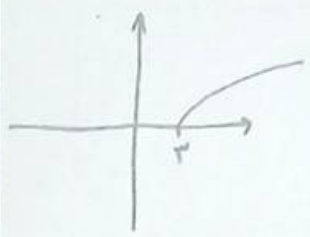


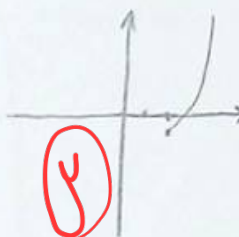
الف) $n=2$ $n^2 - n = 2$ $n^2 - n - 2 = 0 \rightarrow (n-2)(n+1)$ $n=2$ ✓ $n=-1 \rightarrow \times$	ب) $n=2$ $(a, 1) = (a+2, n)$ $a = -1$ $a+2 = 1$ (1) $(1, n) \rightarrow n=2$ ✓	۱
$f(n) = \begin{cases} 2n-1 & n \geq 1 \\ n+a & n < 1 \end{cases}$ 	برای تابع یک به یک باشد باید $n+a < 2$ $\begin{cases} n < 1 \\ n+a < 2 \end{cases} \Rightarrow a < 1$ (1)	۲
$f(n) = \begin{cases} 2n-1 & n \geq 1 \\ an+a-1 & n < 1 \end{cases}$ 	چون می‌خواهیم تابع یک به یک باشد باید شیب (a) مثبت باشد. $a > 0$ (I) $a-1 < 2$ $a < 3$ (II) $(I), (II) \Rightarrow 0 < a < 3$ ✓ (۲)	۳
الف) $y = x^2 + 2$  $n = y^2 + 2$ $n-2 = y^2$ $f^{-1}(n) = \sqrt{n-2}$ ✓	ب) $y = n^2 - 2n + 2$  تابع یک به یک ✓ (۳)	۴
الف) $y = \frac{2n+1}{n-2}$  یک به یک است $n = \frac{2y-1}{2-y}$ $f^{-1}(n) = \frac{2n-1}{2-n}$ ✓ (۱)	ب) $y = \frac{2+2n}{n+2} = 2$ $n \neq -2$  یک به یک است $n = \frac{2-2y}{2-y}$ $f^{-1}(n) = \frac{2-2n}{2-n}$ تابع یک به یک نیست	۵

الف) $y = \sqrt{x-2}$ ع.ک. بدست می آید



$m = \sqrt{y-2}$
 $m^2 = y-2$
 $y = m^2 + 2$
 $f^{-1}(m) = m^2 + 2$ ✓

ب) $y = m^2 - 2m + 2, m \in [2, +\infty)$ ع.ک. بدست می آید



$y = (m-2)^2 - 1$
 $m-2 = \pm \sqrt{y+1}$
 $m = 2 \pm \sqrt{y+1}$
 $f^{-1}(y) = 2 + \sqrt{y+1}$ ✓

$f(m) = m + \sqrt{m}$ $\sqrt{m} = t \Rightarrow y = t^2 + t \Rightarrow t^2 + t - y = 0$
 $t = \frac{-1 \pm \sqrt{1+4y}}{2}$ $\text{بدست می آید} \Rightarrow t = \frac{-1 + \sqrt{1+4y}}{2} \Rightarrow m = \left(\frac{-1 + \sqrt{1+4y}}{2}\right)^2$ ✓
 $m = \frac{1 + 1 + 4y - 2\sqrt{1+4y}}{4} \Rightarrow y = \frac{4m + 2 - 2\sqrt{1+4m}}{4}$
 $f^{-1}(m) = am + b - \sqrt{m-c}$ $\Rightarrow a=1, b=\frac{1}{4}, c=-\frac{1}{4}$ ✓
 $a+2b+2c = 1 + \frac{1}{2} - \frac{1}{2} = 1$ ✓

$y = \frac{m}{m^2-2} \Rightarrow \frac{m_1^2}{m_1^2-2} = \frac{m_2^2}{m_2^2-2} \Rightarrow m_1^2 m_2^2 - 2m_1^2 = m_2^2 m_1^2 - 2m_2^2$
 $m_1^2 = m_2^2 \rightarrow m_1 = \pm m_2$ ✓ *بدست می آید*
 $n = \frac{y}{\sqrt{y^2-2}} \Rightarrow n^2 = \frac{y^2}{y^2-2} \rightarrow y^2 n - 2n^2 = y^2 = y^2 n - y^2 = 2n^2$
 $y^2 = \frac{2n^2}{n^2-1} \rightarrow y = \pm \sqrt{\frac{2n^2}{n^2-1}} = \frac{\sqrt{2n}}{\sqrt{n^2-1}}$ ✓

$f^{-1}(n) = \frac{n}{1+|n|}$ $\text{بدست می آید} \Rightarrow f(x) = \frac{n}{1-|x|} \sim f(-\frac{x}{2}) = \frac{-\frac{x}{2}}{1-|\frac{-x}{2}|} = \frac{-\frac{x}{2}}{\frac{2}{2}-\frac{|x|}{2}} = \frac{-\frac{x}{2}}{\frac{2-|x|}{2}} = -\frac{x}{2-|x|}$
 $g(m) = \sqrt{m} - 1 \sim y = m^2 + 2m + 1 \rightarrow g^{-1}(-\frac{1}{2}) = \frac{1}{4\Delta}$ (II)
 $y = \sqrt{m} - 1$
 $y+1 = \sqrt{m}$
 $y^2 + 2y + 1 = m$
 $(I), (II) \Rightarrow -\frac{1}{2} + \frac{1}{4\Delta} = \frac{-2\Delta}{4\Delta}$ ✓

ج) $f^{-1}(m) = \sqrt{m-1} \rightarrow y = \sqrt{m-1} \sim m = \sqrt{y-1} \Rightarrow m^2 = y-1 \Rightarrow (m^2+1=y)$
 $f(m) = m^2 + 1$ $g(m) = f(m) + \sqrt{f(m)} \xrightarrow{\text{بدست می آید}} g^{-1}(n) = f(m) + \frac{1}{\sqrt{f(m)}} - \frac{1}{\sqrt{f(m)}}$
 $g^{-1}(12) = m^2 + 1 + \frac{1}{\sqrt{m^2+1}} - \frac{1}{\sqrt{m^2+1}} = 12 \Rightarrow m^2 + 1 = 12 \Rightarrow m^2 = 11 \Rightarrow m = \pm \sqrt{11}$
 $g^{-1}(12) = a \rightarrow g(a) = 12 \xrightarrow{g(x) = f(x) + \sqrt{f(x)}} f(a) + \sqrt{f(a)} = 12$
 $g^{-1}(12) = 2$

$f(a) + \sqrt{f(a)} - 12 = 0 \xrightarrow{\sqrt{f(a)} = t} t^2 + t - 12 = 0$
 $(t+4)(t-3) = 0 \rightarrow \begin{cases} \sqrt{f(a)} = 3 \rightarrow f(a) = 9 \rightarrow f^{-1}(9) = a \\ \sqrt{f(a)} = -4 \end{cases}$