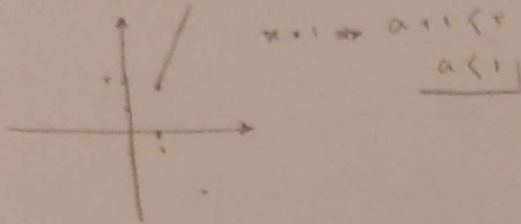
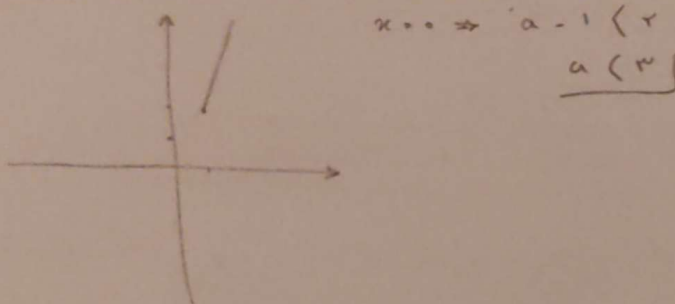


الف) $n^2 - n = 2 \Rightarrow n^2 - n - 2 = 0 \Rightarrow (n-2)(n+1) = 0 \Rightarrow n = 2$ یا $n = -1$ ✓
 ب) $n = 2 \Rightarrow a = 1 \Rightarrow a = 2$

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الف) $y = x^3 + 2$
 $y_1 = x_1^3 + 2$
 $y_2 = x_2^3 + 2$
 $y_1 = y_2 \Rightarrow x_1 = x_2$ یک به یک است.
 $x = y^3 + 2 \Rightarrow y = \sqrt[3]{x-2}$
 ب) $y = x^3 - 2x + 2$
 $\frac{-b}{2a} = \frac{2}{2} = 1 \Rightarrow x = 1$
 $f(1) = 1 - 2 + 2 = 1$

 یک به یک نیست.

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الف) $y = \frac{x+1}{x-2}$

 یک به یک است.
 $x = \frac{y+1}{y-2}$
 $xy - 2x = y + 1 \Rightarrow xy - y = 2x + 1 \Rightarrow y(x-2) = 2x+1$
 $y = \frac{2x+1}{x-2}$
 ب) $y = \frac{2x+1}{x-2}$ یک به یک است.
 $x = \frac{y+1}{y-2}$
 $xy - 2x = y + 1 \Rightarrow xy - 2y = 2x + 1 \Rightarrow y(x-2) = 2x+1$
 $y = \frac{2x+1}{x-2}$
 $y(x-2) = 2x+1 \Rightarrow yx - 2y = 2x+1 \Rightarrow yx - 2x = 2y+1 \Rightarrow x(y-2) = 2y+1$
 $x = \frac{2y+1}{y-2}$

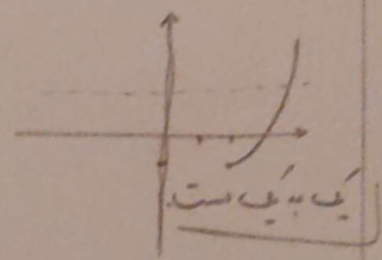
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الف) $y = \sqrt{x+1}$ یک به یک است. $x = \sqrt{y-1}$ $x^2 = y-1$ $y = x^2 + 1$ $D = [0, \infty)$

ب) $y = x^2 - tx + 1$ $\frac{-b}{2a} = \frac{t}{2} = 1$ $t = 2$ $x = 1$

$x = y^2 - ty + 1$ $y^2 - ty + t - 1 = x$ $(y-1)^2 = x+1$

$y-1 = \sqrt{x+1}$ $y = 1 + \sqrt{x+1}$



$y = x + \sqrt{x}$ $x = y + \sqrt{y}$ $a = \sqrt{y}$ $x = a^2 + a$ $a^2 + a - x = 0$

$a = \frac{-1 \pm \sqrt{1+4x}}{2}$ $a = \frac{-1 + \sqrt{4x+1}}{2} = \sqrt{y} \Rightarrow y = \frac{1 + t + t^2 - 2 + t + t^2}{4}$

$= \frac{t^2 + 2t - 1 + t + t^2}{4} = x + \frac{1}{4} - \frac{1}{4}\sqrt{4x+1} = x + \frac{1}{4} - \sqrt{x + \frac{1}{4}}$ $a = 1$ $b = \frac{1}{4}$ $c = -\frac{1}{4}$

$\Rightarrow a + b + c = 1 + 1 - 1 = 1$

$y_1 = \frac{x_1}{\sqrt{x_1^2 - 1}}$
 $y_2 = \frac{x_2}{\sqrt{x_2^2 - 1}}$
 $\left\{ \begin{array}{l} y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2 - 1}} = \frac{x_2}{\sqrt{x_2^2 - 1}} \Rightarrow \frac{x_1^2}{x_1^2 - 1} = \frac{x_2^2}{x_2^2 - 1} \Rightarrow \frac{x_1^2}{x_2^2} = \frac{x_1^2 - 1}{x_2^2 - 1} \Rightarrow x_1^2(x_2^2 - 1) = x_2^2(x_1^2 - 1) \Rightarrow x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2 \Rightarrow -x_1^2 = -x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \end{array} \right.$

$x_1^2(x_2^2 - 1) = x_2^2(x_1^2 - 1)$ $x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2$ $x_1^2 = x_2^2$ $x_1 = \pm x_2$ یک به یک است

$x = \frac{y}{1+y}$ $y > 0$ $x = \frac{y}{y+1}$ $xy + x - y = 0$ $y(x-1) = -x$ $y = -\frac{x}{x-1} = \frac{x}{1-x}$ $0 < x < 1$
 $y < 0$ $x = \frac{y}{1-y}$ $x - xy - y = 0$ $y(x+1) = x$ $y = \frac{x}{x+1}$ $\frac{x}{x+1} < 0$
 $\frac{-1}{-1-x} = -1 < x < 0$ $\Rightarrow f(x) = \begin{cases} \frac{x}{1-x} & 0 \leq x < 1 \\ \frac{x}{x+1} & -1 < x < 0 \end{cases}$

$x = \sqrt{y} - 1$ $x+1 = \sqrt{y}$ $y = (x+1)^2$ $[-1, \infty)$ $f(\frac{1}{2}) + g^{-1}(-\frac{5}{2}) = \frac{\frac{1}{2}}{1-\frac{1}{2}} + (\frac{5}{2})^2 - \frac{5}{2} = \frac{1}{1} + \frac{25}{4} - \frac{5}{2} = \frac{20}{4} + \frac{25}{4} - \frac{10}{4} = \frac{35}{4}$

$x = \sqrt{y} - 1$ $x^2 = y - 1$ $y = x^2 + 1 = f(x)$ $y(x) = x^2 + 1 + \sqrt{x^2 + 1}$

$g^{-1}(x) = y^2 + 1 + \sqrt{y^2 + 1} = x$ $t^2 + 1 + \sqrt{t^2 + 1} = x$ $t = \frac{-1 \pm \sqrt{1+4x}}{2}$ $t = \frac{-1 + \sqrt{4x+1}}{2}$

$\sqrt{y^2 + 1} = \frac{-1 + \sqrt{4x+1}}{2}$ $x = 12 \Rightarrow \frac{-1 + \sqrt{48+1}}{2} = \frac{c}{2} = 4 = \sqrt{y^2 + 1}$

$y^2 + 1 = 16$ $y^2 = 15$ $y = \sqrt{15}$