

(1)

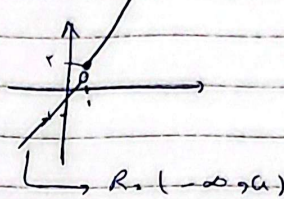
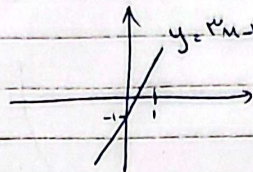
الف) $f = \{(2, 2), (n, 5), (2, n^2 - 1), (2, 2), (-1, 2)\}$

(I) $m = 2 \rightarrow n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow n = 2, -1$

ب) $f = \{(1, 1), (1, 2), (2, 2), (a, a-1), (a+2, n), (n, 2)\}$

(I) $m = 2 \rightarrow a = -1 \rightarrow n = 2$

$f(x) = \begin{cases} x-1 & ; x > 1 \\ x+a & ; x < 1 \end{cases}$



$R_1 \cap R_2 = \emptyset$

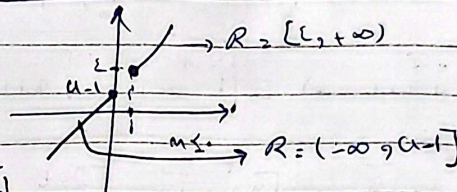
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$a < 2$

$f(x) = \begin{cases} x-1 & ; x > 1 \\ a x + a - 1 & ; x < 1 \end{cases}$

$a \neq 0, a \neq 1 \rightarrow a < 0$

$R_1 \cap R_2 = \emptyset$



if $R_1 \cap R_2 = \emptyset \rightarrow a-1 < 2 \rightarrow a < 3$

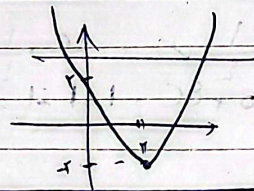
$\begin{cases} a < 3 \\ a > 0 \end{cases} \rightarrow 0 < a < 3$

الف) $y = x^2 + 2 \rightarrow y_1 = x_1^2 + 2 \rightarrow m_1 = x_1 \rightarrow x_1^2 + 2 = x_2^2 + 2 \rightarrow m_1^2 = x_2^2$

$\rightarrow m_1^2 = x_2^2 \rightarrow m_1 = x_2 \leftarrow \text{باید باشد} / x_2 = \sqrt{y_2 - 2} \rightarrow y_2 = \sqrt{m_2 - 2}$

$f^{-1}(m) = \sqrt{m-2}$

ب) $y = x^2 - 4x + 2$



این تابع در بازه $[2, \infty)$ صعودی است و در بازه $(-\infty, 2]$ نزاعی است.

الف) $y = \frac{x+1}{x-2}$



تابع در بازه $(-\infty, 2)$ نزاعی است و در بازه $(2, \infty)$ صعودی است.

$\rightarrow \frac{x+1}{x-2} = y \rightarrow \frac{-y-1}{-y-2} = \frac{x+1}{y+2} = m \rightarrow y = \frac{x+1}{m+2}$

$f^{-1}(m) = \frac{x+1}{m+2}$

ارادہ معلوم (1) $y = \frac{x+2}{x+3} \rightarrow \frac{y(x+3)}{x+3} = \frac{x+2}{x+3} \rightarrow y(x+3) = x+2 \rightarrow yx+3y = x+2 \rightarrow yx-x = 2-3y \rightarrow x(y-1) = 2-3y \rightarrow x = \frac{2-3y}{y-1}$

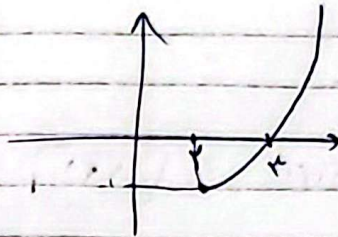
الف) $y = \sqrt{x-3} \rightarrow$



تبع یکتا است

$y = \sqrt{x-3} \rightarrow x = y^2 + 3 \rightarrow x^2 = y^2 + 3 \rightarrow y = \sqrt{x^2 - 3} \rightarrow f^{-1}(x) = \sqrt{x^2 - 3}$

ب) $y = x^2 - 2x + 3 \rightarrow [2, +\infty)$



تبع یکتا است

$y = x^2 - 2x + 3 \rightarrow y = (x-1)^2 + 2 \rightarrow y-2 = (x-1)^2 \rightarrow x-1 = \sqrt{y-2} \rightarrow x = \sqrt{y-2} + 1$

$\rightarrow y = \sqrt{x+1} + 2 \Rightarrow f^{-1}(x) = \sqrt{x+1} + 2$

$f(x) = x + \sqrt{x} = (\sqrt{x})^2 + \sqrt{x} \quad \sqrt{x} = t \rightarrow y = t^2 + t \rightarrow y + t^2 + t = 0$

$\rightarrow t^2 + t - y = 0 \rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2}$

$t = \sqrt{x} \rightarrow x = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2 = \frac{(\sqrt{1+4y} - 1)^2}{4} \rightarrow f^{-1}(x) = \frac{(\sqrt{1+4x} - 1)^2}{4}$

$f^{-1}(x) = \frac{(\sqrt{1+4x} - 1)^2}{4} = x + \frac{1}{x} - \frac{1}{x}(\sqrt{1+4x}) = ax + b - \sqrt{x-c}$

$\rightarrow \boxed{a=1}, \boxed{b=\frac{1}{4}}, \frac{1}{4}\sqrt{1+4x} = \sqrt{x-c} \rightarrow x-c = \frac{1}{4}(1+4x)$

$\rightarrow x-c = \frac{1}{4} + x \rightarrow -c = \frac{1}{4} \rightarrow \boxed{c = -\frac{1}{4}}$

$\rightarrow a=1, b=\frac{1}{4}, c=-\frac{1}{4} \rightarrow ax+2b+4c = 1+1-1 = 1$

الف) $y = \frac{x}{\sqrt{x^2-2}} \rightarrow y = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty) \rightarrow$ تبع یکتا است

صنف
رابطه یکتا و معکوس است
رابطه یکتا و معکوس است

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow y^2 = \frac{x^2}{x^2-2} \rightarrow y^2(x^2-2) = x^2 \rightarrow y^2x^2 - 2y^2 = x^2 \rightarrow y^2x^2 - x^2 = 2y^2 \rightarrow x^2(y^2-1) = 2y^2$

$\rightarrow x^2 = \frac{2y^2}{y^2-1} \rightarrow x = \frac{\sqrt{2}y}{\sqrt{y^2-1}} \rightarrow f^{-1}(x) = \frac{\sqrt{2}y}{\sqrt{y^2-1}}$

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$$f^{-1}(m) = \frac{m}{1+m} \quad \text{پارامیتر} \quad \text{L} \quad x \neq 0 \rightarrow f^{-1}(m) = \frac{x}{x+1} \rightarrow y \cdot m + y = m$$

$$y \cdot m - m = -y \rightarrow m(y-1) = -y \rightarrow m = \frac{-y}{y-1}$$

$$\boxed{f(m) = \frac{m}{1+m}} \quad \leftarrow f(m) = \frac{-m}{x-1} \quad \leftarrow y = \frac{-m}{m-1}$$

$$g(m) = \sqrt{m-1} \rightarrow y = \sqrt{m-1} \rightarrow \sqrt{y} + 1 = x \rightarrow \sqrt{y} = m+1 \rightarrow y = (m+1)^2$$

$$\rightarrow \boxed{g^{-1}(m) = (m+1)^2} \quad \text{خاصیت دراز} \quad f\left(-\frac{r}{8}\right) + g^{-1}\left(-\frac{r}{8}\right)$$

$$\rightarrow f\left(-\frac{r}{8}\right) = \frac{-\frac{r}{8}}{1-\frac{r}{8}} = \frac{-\frac{r}{8}}{\frac{8-r}{8}} = \frac{-r}{8-r} = \frac{-r}{8} \quad , \quad g^{-1}\left(-\frac{r}{8}\right) = \left(-\frac{r}{8} + 1\right)^2 = \frac{4}{8}$$

$$\rightarrow f\left(-\frac{r}{8}\right) + g^{-1}\left(-\frac{r}{8}\right) = \frac{-r}{8} + \frac{4}{8} = \frac{-r+4}{8} = \frac{-24}{8}$$

$$f^{-1}(m) = \sqrt[3]{m-1} \rightarrow y = \sqrt[3]{m-1} \rightarrow x = \sqrt[3]{y-1} \rightarrow x^3 = y-1 \quad 10$$

$$\rightarrow x^3 + 1 = y \rightarrow y = \boxed{f(m) = x^3 + 1} \quad , \quad g(m) = f(m) + \sqrt{f(m)}$$

$$\rightarrow g(m) = x^3 + 1 + \sqrt{x^3 + 1} \quad x^3 + 1 = t \quad y = t^2 + t \rightarrow t^2 + t - y = 0$$

$$\Delta = t = \frac{-1 \pm \sqrt{1+4y}}{2} \quad t \geq 0 \quad , \quad t = \frac{-1 + \sqrt{1+4y}}{2} \rightarrow x^3 + 1 = t^2 \rightarrow x^3 = t^2 - 1$$

$$\rightarrow x^3 = t^2 - 1 = \left(\frac{-1 + \sqrt{1+4y}}{2}\right)^2 - 1 = \frac{1 - 2\sqrt{1+4y} + (1+4y)}{4} = \frac{2+4y-2\sqrt{1+4y}}{4}$$

$$= \frac{1+2y-\sqrt{1+4y}}{2} \rightarrow x^3 = \frac{2y+1-\sqrt{1+4y}}{2} \quad \boxed{\frac{2y+1-\sqrt{1+4y}}{2} = x^3}$$

$$\rightarrow g^{-1}(m) = \sqrt[3]{\frac{2m+1-\sqrt{1+4m}}{2}} \quad \text{خاصیت دراز} \quad g^{-1}(14) = \sqrt[3]{\frac{28-1-\sqrt{1+28}}{2}} = \sqrt[3]{\frac{27-5}{2}}$$

$$= \sqrt[3]{\frac{22-5}{2}} = \sqrt[3]{\frac{17}{2}} = \sqrt[3]{8.5} = 2 \rightarrow \text{پایان}$$