

(1)

الف) $f = \{(2, 2), (n, 5), (2, n^2 - n), (2, 2), (-1, 2)\}$

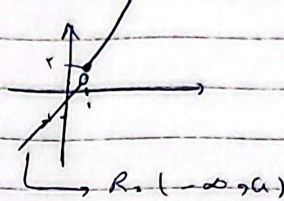
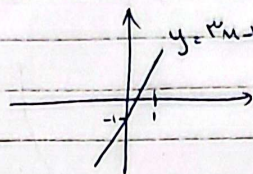
(I) $m = 2 \rightarrow n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow n = 2, -1$ ✓

ب) $f = \{(1, 1), (1, 2), (2, 2), (a, n-1), (a+2, n), (n, 2)\}$

(I) $m = 2 \rightarrow a = -1 \rightarrow n = 2$ ✓

(2)

$f(x) = \begin{cases} x-1 & x > 1 \\ x+a & x < 1 \end{cases}$



(1)

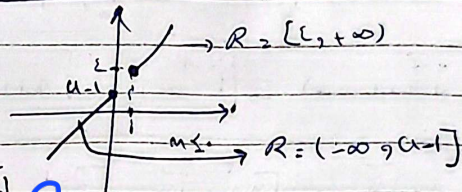
$R_1 \cap R_2 = \emptyset$

~~scribbled out text~~

$a < 2$

$1+a < 2$
 $a < 1$

$f(x) = \begin{cases} x-1 & x > 1 \\ a x + a - 1 & x < 1 \end{cases}$



$a > 0$

$0 < a < 1$

$R_1 \cap R_2 = \emptyset$

$a-1 < 2$

$a < 3$

if $R_1 \cap R_2 = \emptyset \rightarrow a-1 < 2 \rightarrow a < 3$

$\begin{cases} a < 1 \\ a > 0 \end{cases} \rightarrow 0 < a < 1$

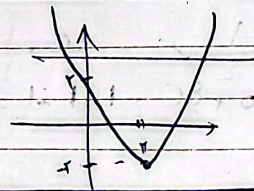
الف) $y = x^2 + 2 \rightarrow y_1 = 2 \rightarrow m_1 = x_1 \rightarrow x_1^2 + 2 = x_1^2 + 2 \rightarrow m_1^2 = x_1^2$

$\rightarrow m_1^2 = x_1^2 \rightarrow m_1 = x_1 \leftarrow \text{باید باشد} / x^2 = y^2 \rightarrow m = \sqrt{y-2} \rightarrow y = \sqrt{m-2}$

(2)

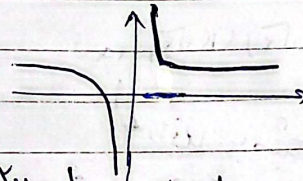
$f^{-1}(m) = \sqrt{m-2}$ ✓

ب) $y = x^2 - 4x + 2$



این تابع در بازه $[2, \infty)$ صعودی است و در بازه $(-\infty, 2]$ نزاعی است. ✓

الف) $y = \frac{x+1}{x-2}$



تابع هذلولی است پس باید یکدست

$\rightarrow \frac{x+1}{x-2} = y \rightarrow \frac{-y-1}{-y-2} = \frac{x+1}{x-2} = m \rightarrow y = \frac{x+1}{x-2}$

$f^{-1}(m) = \frac{x+1}{x-2}$

1/10

ارادہ معلوم () اس میں زیر غور ہے۔ یا زخم قوی ہے۔
 یہ بہ یک نسبت $\rightarrow y = 2 \rightarrow$ تابع ثابت $\rightarrow \frac{2(x+2)}{x+2} = 2 \rightarrow$ $y = \frac{x+2}{x+2}$ (ب)

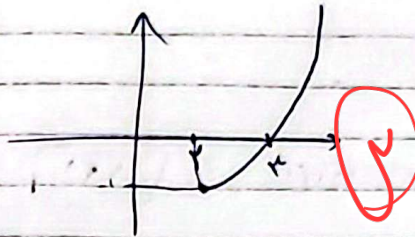
(الف) $y = \sqrt{x-3} \rightarrow$



تابع یک بہ یک است

$y = \sqrt{x-3} \rightarrow x = \sqrt{y-3} \rightarrow x^2 = y-3 \rightarrow y = x^2+3 \rightarrow f^{-1}(x) = x^2+3$

(ب) $y = x^2 - 2x + 3 \rightarrow [2, +\infty)$



تابع یک بہ یک است

$y = x^2 - 2x + 3 \rightarrow y = (x-2)^2 - 1 \rightarrow y+1 = (x-2)^2 \rightarrow x-2 = \sqrt{y+1} \rightarrow x = \sqrt{y+1} + 2$

$\rightarrow y = \sqrt{x+1} + 2 \Rightarrow f^{-1}(x) = \sqrt{x+1} + 2$

(۷) $f(x) = x + \sqrt{x} = (\sqrt{x})^2 + \sqrt{x}$ $\sqrt{x} = t \rightarrow y = t^2 + t \rightarrow y + t^2 + t = 0$

$\rightarrow t^2 + t - y = 0 \rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2}$ $\frac{-1 + \sqrt{1+4y}}{2}$ (موجب)

$t = \sqrt{x} \rightarrow x = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2 = \frac{(\sqrt{1+4y} - 1)^2}{4} \rightarrow f^{-1}(x) = \frac{(\sqrt{1+4x} - 1)^2}{4}$

$f^{-1}(x) = \frac{(\sqrt{1+4x} - 1)^2}{4} = x + \frac{1}{x} - \frac{1}{x}(\sqrt{1+4x}) = ax + b - \sqrt{x-c}$

$\rightarrow \boxed{a=1}, \boxed{b=\frac{1}{x}}, \frac{1}{x}\sqrt{1+4x} = \sqrt{x-c} \rightarrow x-c = \frac{1}{4}(1+4x)$

$\rightarrow x-c = \frac{1}{4} + x \rightarrow -c = \frac{1}{4} \rightarrow \boxed{c = -\frac{1}{4}}$
 $\rightarrow a=1, b=\frac{1}{x}, c=-\frac{1}{4} \rightarrow ax+by+cz = 1+1-1 = 1$ $\rightarrow$ خالصہ ۱

(۸) $y = \frac{x}{\sqrt{x^2-2}} \rightarrow y = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$ $\rightarrow$ یک بہ یک

مفروضہ x, y صحیح اعداد ہیں۔
 دو صحیح اعداد کا تناسب

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow y^2 = \frac{x^2}{x^2-2} \rightarrow y^2(x^2-2) = x^2 \rightarrow y^2x^2 - 2y^2 = x^2 \rightarrow y^2x^2 - x^2 = 2y^2 \rightarrow x^2(y^2-1) = 2y^2$
 $\rightarrow x^2 = \frac{2y^2}{y^2-1} \rightarrow x = \frac{\sqrt{2}y}{\sqrt{y^2-1}} \rightarrow f^{-1}(x) = \frac{\sqrt{2}y}{\sqrt{y^2-1}}$

$$f^{-1}(m) = \frac{m}{1+m} \quad \text{با فرض } f(m) = \frac{m}{1+m} \quad \text{①} \quad x \neq -1 \rightarrow f^{-1}(m) = \frac{x}{x+1} \rightarrow y = m \rightarrow y = \frac{x}{x+1}$$

$$y(x+1) = x \rightarrow yx + y = x \rightarrow yx - x = -y \rightarrow x(y-1) = -y \rightarrow x = \frac{-y}{y-1}$$

$$\boxed{f(m) = \frac{m}{1+m}} \quad \leftarrow f(m) = \frac{-m}{x-1} \quad \leftarrow y = \frac{-m}{m-1}$$

$$g(m) = \sqrt{m-1} \rightarrow y = \sqrt{m-1} \rightarrow \sqrt{y^2+1} = x \rightarrow \sqrt{y} = m+1 \rightarrow y = (m+1)^2$$

$$\rightarrow \boxed{g^{-1}(m) = (m+1)^2} \quad \text{حاصل نهایی} \quad f\left(-\frac{r}{8}\right) + g^{-1}\left(-\frac{r}{8}\right)$$

$$\rightarrow f\left(-\frac{r}{8}\right) = \frac{-\frac{r}{8}}{1-\frac{r}{8}} = \frac{-\frac{r}{8}}{\frac{8-r}{8}} = \frac{-r}{8-r} = \frac{-r}{8} \quad , \quad g^{-1}\left(-\frac{r}{8}\right) = \left(-\frac{r}{8}+1\right)^2 = \frac{4}{8}$$

$$\rightarrow f\left(-\frac{r}{8}\right) + g^{-1}\left(-\frac{r}{8}\right) = \frac{-r}{8} + \frac{4}{8} = \frac{-r+4}{8} = \frac{-24}{8} = -3 \quad \checkmark \quad \text{②}$$

$$f^{-1}(m) = \sqrt[3]{m-1} \rightarrow y = \sqrt[3]{x-1} \rightarrow x = \sqrt[3]{y^3+1} \rightarrow x^3 = y^3+1 \quad \text{③}$$

$$\rightarrow x^3+1 = y^3 \rightarrow y = \sqrt[3]{x^3+1} \quad , \quad g(m) = f(m) + \sqrt{f(m)}$$

$$\rightarrow g(m) = x^3+1 + \sqrt{x^3+1} \quad x^3+1 = t \quad y = t^2+t \rightarrow t^2+t-y=0 \quad \text{④}$$

$$\Delta = t = \frac{-1 \pm \sqrt{1+4y}}{2} \quad t \geq 0 \quad , \quad t = \frac{-1 + \sqrt{1+4y}}{2} \rightarrow x^3+1 = t^2 \rightarrow x^3 = t^2-1$$

$$\rightarrow x^3 = t^2-1 = \left(\frac{-1 + \sqrt{1+4y}}{2}\right)^2 - 1 = \frac{1 - 2\sqrt{1+4y} + (1+4y)}{4} = \frac{2+4y-2\sqrt{1+4y}}{4}$$

$$= \frac{1+2y-\sqrt{1+4y}}{2} \rightarrow x^3 = \frac{2y+1-\sqrt{1+4y}}{2} \quad \boxed{\frac{2y+1-\sqrt{1+4y}}{2} = x^3}$$

$$\rightarrow g^{-1}(m) = \sqrt[3]{\frac{2m+1-\sqrt{1+4m}}{2}} \quad \text{حاصل نهایی} \quad g^{-1}(14) = \sqrt[3]{\frac{28-1-\sqrt{1+56}}{2}} = \sqrt[3]{\frac{27-8}{2}} = \sqrt[3]{9}$$

$$= \sqrt[3]{\frac{27-8}{2}} = \sqrt[3]{9} = \sqrt[3]{27} = 3 \quad \checkmark \quad \text{پایان}$$