

الف) $f = \{(2, 2), (h, 2), (2, h^2 - h), (m, 2), (-1, 4)\}$

-1

$(2, 2), (2, h^2 - h) \Rightarrow h^2 - h = 2 \Rightarrow h^2 - h - 2 = 0$ $(m, 2) = (2, 2) \Rightarrow m = 2$ $h = 2$

$(h - 2)(h + 1) = 0 \Rightarrow h = 2$ $(-1, 4)$

ب) $f = \{(-1, 1), (1, 2), (2, 3), (m, 2), (2, h), (m, 2)\}$

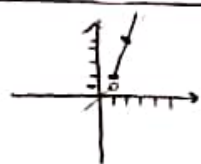
$(2, 3) = (m, 2) \Rightarrow m = 2$

$(1, 2) = (2, h) \Rightarrow h = 2$

$(-1, 1) = (a, 1) \Rightarrow a = -1$

$(1, 2) = (1, h) \Rightarrow h = 2$

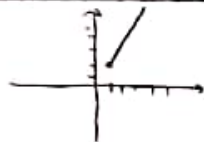
$f(x) = \begin{cases} x - 1 & x \geq 1 \\ x + a & x < 1 \end{cases}$



$x + a < 2 \Rightarrow a < 2 - x$ $a \leq 1$

-2

$f(x) = \begin{cases} x - 1 & x \geq 1 \\ a x + a - 1 & x < 1 \end{cases}$



$a x + a - 1 < 2$

$\begin{cases} a > 0 \Rightarrow x = 0 \Rightarrow a - 1 < 2 \Rightarrow a < 3 \\ a = 0 \Rightarrow -1 < 2 \Rightarrow a < 2 \\ a < 0 \Rightarrow x = 0 \Rightarrow -a - 1 < 2 \Rightarrow -a < 3 \Rightarrow a > -3 \end{cases}$

$a = (-3, 3)$

-3

الف) $y = x^3 + 2$

$y - 2 = x^3$

$x = \sqrt[3]{y - 2}$

لاصقوس

$y = \sqrt[3]{x - 2}$



✓

ب) $y = x^2 - 4x + 2$

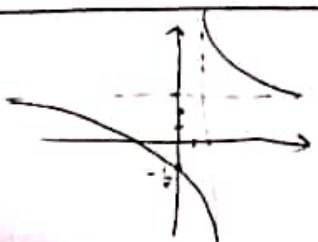
✗

$\Delta = 16 - 8 = 8 \Rightarrow \Delta > 0$ دارد دو ریشه حقیقی

-4

الف) $y = \frac{x + 1}{x - 2}$

تابع هذلولی
شکلوس بدید



✓

ب) $y = \frac{x + 2}{x + 1}$

$\frac{x + 2}{x + 1} = 2$

تابع ثابت

پس یک یکتا نیست

-5

$x + 1 = y(x - 2)$

$x(1 - y) = -1 - 2y$

$x = \frac{-1 - 2y}{1 - y} = \frac{2y + 1}{y - 1}$

$x = \frac{2y + 1}{y - 1} \Rightarrow y = \frac{x + 1}{x - 2}$

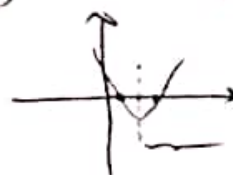
الف) $y = \sqrt{x-2}$ یک یک به یک است ✓



معکوس
 $y = \sqrt{x-2} \Rightarrow y^2 = x-2$
 $\Rightarrow \boxed{y = x^2 + 2}$ $x = y^2 + 2$

$y = x^2 - 2x + 2 = (x-1)(x-1)$ - 4

$x \in [2, +\infty)$ پس یک یک به یک است ✓



$y = x^2 - 2x + 2$

$y = (x-1)^2 - 1 \Rightarrow (x-1)^2 = y+1$

$x-1 = \sqrt{y+1}$

$x = \sqrt{y+1} + 1$

$\boxed{y = \sqrt{x-1} + 1}$

$f(x) = x + \sqrt{x} \Rightarrow x > 0$

$y = x + \sqrt{x}$

$x = \frac{y^2 + 1 \pm \sqrt{(y^2 + 1)^2 - 4y^2}}{2}$

$\Rightarrow x = \frac{y^2 + 1 \pm \sqrt{4y^2 + 1}}{2}$

$(x, \sqrt{x}) \Rightarrow 0, \sqrt{0} = 0$

$\Rightarrow a + b + c = 1 + 1 - 1 = 1$

$y - x = \sqrt{x} \Rightarrow (y-x)^2 = x$

$y^2 - 2xy + x^2 = x$

$y^2 + x^2 - 2xy - x = 0$

$x^2 + x(1-y) + y^2 = 0$

$(0, +\infty) \Rightarrow y > 0 \Rightarrow y = \frac{y^2 + 1 - \sqrt{4y^2 + 1}}{2} = a + b - \sqrt{c}$

$\begin{matrix} a & b & c \\ \downarrow & \downarrow & \downarrow \\ x & \frac{1}{2} & -\sqrt{x+1} \end{matrix} \Rightarrow a = 1, b = \frac{1}{2}, c = -1$

(1) - V

$y = \frac{x}{\sqrt{x^2 - 2}} \xrightarrow{y_1 = y_2} \left(\frac{x_1}{\sqrt{x_1^2 - 2}} = \frac{x_2}{\sqrt{x_2^2 - 2}} \right)^2 = \frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2}$ $x = \frac{y}{\sqrt{y^2 - 2}} \Rightarrow \frac{y^2}{y^2 - 2} = x^2$ - 1

$x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \Rightarrow 2x_1^2 = 2x_2^2 \Rightarrow x_1 = \pm x_2$

$x_1 = -x_2 \Rightarrow$ چون اولی و دومی
 $\Rightarrow \frac{x_1}{\sqrt{x_1^2 - 2}} = \frac{x_2}{\sqrt{x_2^2 - 2}} \Rightarrow x_1 = x_2 \Rightarrow y^2 = \frac{x^2}{x^2 - 1} \Rightarrow \boxed{y = \frac{\sqrt{x^2}}{\sqrt{x^2 - 1}}}$

$f^{-1}(x) = \frac{x}{1+|x|}$

$g(x) = \sqrt{x} - 1$

$\frac{x}{1+|x|} = -\frac{1}{0}$

$\frac{dx}{dt} = -\frac{1}{x^2} \Rightarrow \frac{1}{x^2} = -\frac{1}{x^2} \Rightarrow x < 0$

$f\left(-\frac{9}{10}\right) + g^{-1}\left(-\frac{9}{10}\right)$

$\sqrt{x} - 1 = -\frac{9}{10} \Rightarrow \sqrt{x} = \frac{1}{10}$

$\sqrt{x} = -\frac{9}{10} \Rightarrow x = \frac{81}{100}$

$-\frac{9}{10} + \frac{9}{10} = \frac{-9 + 9}{10} = \frac{0}{10} = 0 \Rightarrow \boxed{g^{-1}\left(-\frac{9}{10}\right) = \frac{9}{10}}$

$f^{-1}(x) = \sqrt[3]{x-1}$

$g(x) = f(x) + \sqrt{f(x)}$

$g^{-1}(12) = ? = \boxed{2}$

$f(x) + \sqrt{f(x)} = 12 \Rightarrow \boxed{f(x) = 9}$

$f^{-1}(9) = \sqrt[3]{9-1} = \boxed{2}$

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