

$$A = \{1, 2, 3\} \quad B = \{2, 3\} \quad A \times B - B^c \rightarrow ?$$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$\rightarrow A \times B - B^c = \{(1, 2), (1, 3)\}$$

$$f(x) = \{(2, m^2), (2, 1), (-2, m), (-2, m), (2, m+2), (m, 4)\}$$

$$(2, m^2) = (2, m+2) \rightarrow m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) \rightarrow m = -1, 2$$

$$\text{if } m = 2 \rightarrow (2, 1) \neq (2, 4) \rightarrow m \neq 2 \rightarrow \boxed{m = -1}$$

$$g(x) = \{(2, m^2), (2, 1), (m, 4), (2, m+2), (-2, m), (-1, 2)\}$$

$$m^2 = m+2 \rightarrow m = -1, 2 \xrightarrow{\text{if}} m = -1 \rightarrow (-1, 4) \neq (-1, 2) \xrightarrow{\text{if}} m = 2 \rightarrow (2, 1) \neq (2, 4)$$

$$f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \\ ax + b & ; 1 \leq x \leq r \\ x^r & ; x \geq r \end{cases}$$

$$\xrightarrow{x=1} 1 - 1 = a + b \rightarrow a + b = 0$$

$$\xrightarrow{x=r} (r)^r = a(r) + b \rightarrow 1 = 2a + b$$

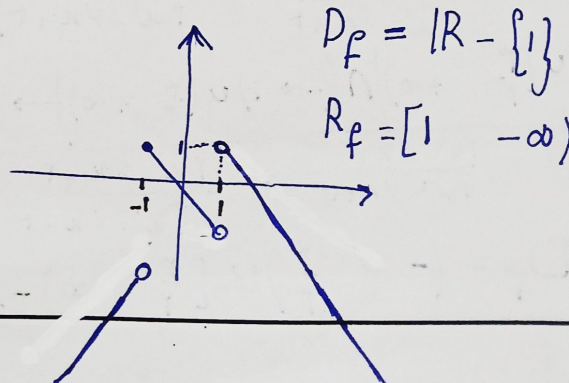
$$\boxed{a = 1, b = -1}$$

$$\boxed{ab = -1} \leftarrow a = 1, b = -1$$

$$f(x) = \begin{cases} 2x - r & ; x \geq k \\ f - 2x & ; x \leq k \end{cases} \rightarrow 2x - r = f - 2x \rightarrow 4x = f$$

$$\rightarrow x = \frac{f}{4} \rightarrow \boxed{k = \frac{f}{4}}$$

$$f(x) = \begin{cases} -x + 1 & ; x > 1 \\ -x & ; -1 \leq x < 1 \\ x - 1 & ; x < -1 \end{cases}$$

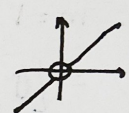


الف) $2y^3 + x^3 + 1 = 0$ تبدیل متغیر $\begin{cases} x_1 \rightarrow y_1^3 \\ x_2 \rightarrow 2y_2^3 \end{cases} \xrightarrow{x_1 = x_2} 2y_1^3 = 2y_2^3 \rightarrow y_1^3 = y_2^3$
 $\rightarrow y_1 = y_2 \rightarrow$ تابع هست ✓

ب) $x^3 + 4y^3 - 2x - 12y + 1 = 0 \rightarrow x^3 - 2x + 1 + 4y^3 - 12y + 9 = 0$
 $\rightarrow (x-1)^3 + (2y-3)^3 = 0 \rightarrow x=1, y=\frac{3}{2}$ یک نقطه $\rightarrow$ تابع هست ✓

الف) $x \sin y = y \sin x$ مثال نقض $x=0^\circ \rightarrow \sin 0^\circ = 0 \rightarrow 0 \times \sin y = y \times 0$
 $\rightarrow y \in \mathbb{R} \rightarrow$ تابع نیست ✗

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$ $\xrightarrow{\frac{x}{y}=t} \sqrt{t} + \sqrt{\frac{1}{t}} = 2$ ضرب $t + \frac{1}{t} + 2 = 2$

$\rightarrow t + \frac{1}{t} - 2 = 0 \xrightarrow{\times t} t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0 \rightarrow t=1 \rightarrow x=y$  تابع هست ✓

الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{2-x}{x}}$

$\rightarrow \frac{x-1}{x-3} \geq 0 \rightarrow \frac{1}{0} - \frac{1}{0} \rightarrow A$

$\rightarrow \frac{2-x}{x} \geq 0 \rightarrow \frac{0}{0} + \frac{2}{0} \rightarrow B$

$D = A \cap B = (0, 1]$

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3} \Rightarrow \sqrt{x} - \sqrt{3} = -\sqrt{y-1}$

① $x \geq 0$

② $\sqrt{x} - \sqrt{3} \leq 0 \rightarrow \sqrt{x} \leq \sqrt{3} \rightarrow x \leq 3$

$\rightarrow D_f = ① \cap ② = [0, 3]$

الف) $\frac{\sqrt{x^2-5x+4}}{\sqrt{x-10x+14}} \rightarrow x^2-5x+4 \geq 0 \rightarrow (x-4)(x-1) \geq 0 \rightarrow \frac{1}{0} - \frac{4}{0} \rightarrow (-\infty, 1] \cup [4, +\infty) \rightarrow A$
 $\frac{\sqrt{x^2-10x+14}}{\sqrt{x-10x+14}} \rightarrow -9x+14 \geq 0 \rightarrow -9x \geq -14 \rightarrow x \leq \frac{14}{9} \rightarrow B$
 $\rightarrow A \cap B \rightarrow (-\infty, 1]$

ب) $\frac{\sqrt{x^2-5x+4}}{x^2-10x+14} \rightarrow \frac{x^2-5x+4}{x^2-10x+14} \geq 0 \rightarrow \frac{(x-4)(x-1)}{(x-1)(x-2)} \geq 0 \rightarrow \frac{1}{0} - \frac{4}{0} + \frac{1}{0} - \frac{1}{0} \rightarrow$
 $\rightarrow D_f = (-\infty, 1] \cup (2, 4] \cup (4, +\infty)$

الف) $y = \frac{\sqrt{2x^2-5x+3}}{\sqrt{2x^2-5x+3}}$ $\rightarrow 2x^2-5x+3 \geq 0 \rightarrow (x-1)(2x-3) \rightarrow \frac{1}{0} - \frac{3}{0}$
 $2x^2-5x+3 > 0 \rightarrow (x-1)(2x-3) \rightarrow \frac{1}{0} - \frac{3}{0}$

$((-\infty, 1] \cup [\frac{3}{2}, +\infty)) \cap ((-\infty, 1) \cup (\frac{3}{2}, +\infty)) \rightarrow D_f = (-\infty, 1) \cup [\frac{3}{2}, +\infty)$

ب) $y = \frac{\sqrt{2x^2-5x+3}}{2x^2-5x+3} \rightarrow \frac{2x^2-5x+3}{2x^2-5x+3} \geq 0 \rightarrow \frac{(x-1)(2x-3)}{(x-1)(2x-3)} \rightarrow \frac{1}{0} + \frac{3}{0} - \frac{1}{0} - \frac{3}{0}$

$\rightarrow D_f = (-\infty, \frac{3}{2}) \cup [\frac{3}{2}, +\infty) - \{1\}$