

$$f \circ f\left(\frac{x}{2}\right) = f\left(f\left(\frac{x}{2}\right)\right)$$

$$f\left(\frac{x}{2}\right) = \cot + \frac{\pi x}{2} \rightarrow \cot + \frac{\pi}{2} \Rightarrow \sqrt{2}$$

$$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = 2$$

$$\text{الف) } f \circ g\left(\frac{\pi}{2}\right) = ? \quad g\left(\frac{\pi}{2}\right) = 2 \cos^2 \pi = 2 \times \frac{1}{2} = 1$$

$$f\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right) \rightarrow 2 = 2 \quad f(2) = \sqrt{\frac{2-1}{2}} = \left(\frac{\sqrt{2}}{2}\right)$$

ب)

$$f \circ g(\sqrt{2}) = ? \quad g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1}$$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -2$$

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{2}} = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = \left(\frac{1}{2}\right)$$

$$\text{الف) } f \circ g(x) = \{(1, a) (2, a) (4, 12) (8, 12)\}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$\text{د) } g \circ g(x) = \{(2, 1) (2, 4) (4, 10)\}$$

$$g(x) = t \rightarrow f(t) = 2 \rightarrow t = 3 \rightarrow x = a \rightarrow a = 2$$

$$f(x) = b \rightarrow (1, a) \rightarrow b = a \rightarrow \boxed{1, a}$$

$$f(x) = rx + 1 \rightarrow g(f(-1)) = \boxed{-1}$$

$$f(x) = -rx - c \rightarrow g(f(-1)) = \boxed{-1}$$

$$f(f(x)) = a(ax+b) + b = ax + ab + b = \varepsilon x + c$$

$$ar = \varepsilon$$

$$a = r \rightarrow b = 1$$

$$a = \pm r$$

$$a = -r \rightarrow b = -c$$

$$g(x) = cx + d$$

$$rc = \varepsilon \rightarrow c = \frac{\varepsilon}{r}$$

$$g(rx+c) = c(rx+c) + d = cx - r \Rightarrow rc + d = -r$$

$$g(x) = cx + d = \frac{\varepsilon}{r}x - \frac{1\varepsilon}{r}$$

$$rc + d = -r$$

$$d = -\frac{1\varepsilon}{r}$$

$$f(x) \begin{cases} \sqrt{x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$D_g = \mathbb{R} - \{0, \varepsilon\}$$

$$D_f = \mathbb{R}$$

$$\sqrt{x+|x|} \neq \{0, \varepsilon\}$$

$$\sqrt{x+|x|} \neq 0 \rightarrow x > 0$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$\sqrt{x+|x|} \neq \varepsilon \rightarrow x \neq 1$$

$$\{x \in D_f, f(x) \in D_{f \circ g}\}$$

$$\sqrt{1-x^2} \in [0, 1] \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 < 1-x^2 \leq 1$$

$$\rightarrow 1 \leq -x^2 \leq 0 \rightarrow 0 \leq x^2 \leq 1 \rightarrow -1 \leq x \leq 1$$

$$D_f = [-1, 1]$$

$$\frac{-1}{-1+1} \quad \frac{1}{-1+1}$$

$$D_{(f+g)} = D_f \cap D_g = [0, 1]$$

$$D_g = [0, +\infty)$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$\text{الف) } \frac{z+1}{z-1} = t$$

$$f(t) = \mathcal{L}\left(\frac{t+1}{t-1}\right) + a$$

$$z = \frac{t+1}{t-1} \rightarrow \boxed{f(z) = \frac{|z-1|}{z-1}}$$

$$\text{ب) } z + \frac{1}{z} = t \quad t^c = z^c + \frac{1}{z^c} + c \left(z + \frac{1}{z} \right)$$

$$\Rightarrow t^c = z^c + \frac{1}{z^c} + ct \Rightarrow z^c + \frac{1}{z^c} = t^c - ct$$

$$f(t) = t^c - ct \quad \boxed{f(z) = z^c - cz}$$

$$z > 0$$

$$\mathcal{L}(f(z)) = 0$$

$$g(z) = 0$$

$$\hookrightarrow z_1 = 1$$

$$\hookrightarrow z_2 = \sqrt{2}$$

$$f(z) \begin{cases} z_1 = 1 & z\sqrt{2} = 1 & z'_1 = 1 \\ z_2 = \sqrt{2} & z\sqrt{2} = \sqrt{2} & z'_2 = 2 \end{cases}$$

$$z'_2 - z'_1 = 2 - 1 = 1$$