Subject: (تکلیف ۹) (یازدهم دفتر) پنجشنبه — Date: (حمامه مختاری)

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①

$f(x)=\begin{cases}\cot\frac{\pi}{4}x, & x\le 1\\ \sqrt{x^2+1}\end{cases}$

$fof(\frac{2}{3})=?$

$f(f(\frac{2}{3}))=?$

$f(\frac{2}{3})=\cot\frac{\pi}{6}\Rightarrow\sqrt{3}$

$f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2$

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②

الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}\Rightarrow g(x)=2\cos^2 x$

$fog(\frac{\pi}{3})=?\qquad g(\frac{\pi}{3})=2\cos^2\frac{\pi}{3}=2\times\frac{1}{4}=\frac{1}{2}$

$f(\frac{1}{x})=f(\frac{1}{2})\to \sqrt{\frac{4-1}{4}}\Rightarrow\frac{\sqrt{3}}{2}$

$x=2$

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ب) $f(x)=[x]\qquad g(x)=\frac{x}{1-x}$ ③

$fog(\sqrt{2})=?\qquad g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{-1}=-2-\sqrt{2}$

$f(-2-\sqrt{2})=[-\sqrt{2}-2]=-4$

$-3/...$

③

$f(x)=\sin x\qquad f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

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$g(x)=x\sqrt{1-x^2}\qquad g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}}=\frac{1}{2}$

$\frac{1}{\sqrt{2}}$

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$f(x) = \{(\varepsilon,0), (4,0), (\Lambda,12), (10,12)\}$ (٤)

$g(x) = \{(\varepsilon,4), (2,\varepsilon), (4,\Lambda), (\Lambda,10)\}$ (٥)

الف) $f \circ g(x) = \{(\varepsilon,0)(2,0)(4,12)(\Lambda,12)\}$

ب) $g \circ f(x) = \emptyset$ ج) $f \circ f(x) = \emptyset$

د) $g \circ g(x) = \{(\varepsilon,\Lambda), (2,4), (4,10)\}$

(٥)

$f = \{(2,1), (3,2), (\varepsilon,0), (1,\Lambda)\}$ (٥)

$g = \{(1,2), (3,1), (a,3), (b,1)\}$

$(\varepsilon,2) \in f \circ g \rightarrow g(x)=t \rightarrow f(t)=2 \rightarrow t=3 \rightarrow x=a \rightarrow a=\varepsilon$

$(\varepsilon,1) \in g \circ f \quad g(f(x))=1 \rightarrow f(x)=b \quad (\varepsilon,b) \rightarrow b=0$

$(a,b) \Rightarrow (\varepsilon,0)$

$f \circ g$ تابع یک به یک $f(x) = 4x+1 \rightarrow g(f(-1)) = -\Lambda$ (٦)

$f(f(x)) = \varepsilon x+3$ $f(x) = -2x-3$ (٥)

$g(2x+2) = 2x-2$ $f(f(x)) = a(ax+b)+b = a^2x+ab+b = \varepsilon x+3$

$g \circ f(-1) = ?$ $a^2 = \varepsilon \Rightarrow \begin{cases} a=2 \rightarrow b=1 \\ a=-2 \rightarrow b=3 \end{cases}$

$g(x) = cx+d$ $2c = 2 \rightarrow c = \frac{3}{2}$

$g(2x+2) = c(2x+2)+d = 3x-2 \Rightarrow 3c+d = -2 \rightarrow d = \frac{-12}{-2}$

$\rightarrow \frac{3}{2}x - \frac{12}{2}$ $2cx+3c+d$

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$f(x)=\sqrt{x+|x|}$ $\quad g(x)=\frac{1}{x^2-4x}$ (۷)

$f(x)\begin{cases}\sqrt{2x} & x>0\\ 0 & x\le 0\end{cases}$ $\quad D_g=R-\{0,4\}$

$D_f=R$ $\quad \sqrt{x+|x|}\neq\{0,4\}$ (۶)

$D_{gof}=?$ $\quad \sqrt{x+|x|}\neq 0 \to x>0$

$\sqrt{x+|x|}\neq 4 \to x\neq 8$ $\quad D_{gof}=(0,+\infty)-\{8\}$

$f(x)=\sqrt{1-x^2}$ $\quad \{x\in D_f,\ f(x)\in D_{fog}\}$ (۸)

$g(x)=\sqrt{x}$ (۶)

$D(fofofogof)=?$ $\quad \sqrt{1-x^2}\in[0,1] \Rightarrow 0\le\sqrt{1-x^2}\le 1$

$-1\le -x^2\le 0 \to 0\le x^2\le 1$ (۱) $\quad$ توجه $\quad 0\le 1-x^2\le 1$

$D_f=\frac{-1\ \ +1}{-\ +\ -}$ $\quad D_f=[-1,1]$ $\quad \boxed{-1\le x\le 1}$ (۲)

$D_g=[0,+\infty)$ $\quad D_{(f+g)}=D_f\cap D_g=[0,+1]$

الف) $f\left(\frac{2x+1}{x-2}\right)=4x+9$

$f(x)=?$ $\quad \frac{2x+1}{x-2}=t$

$f(t)=4\left(\frac{2t+1}{t-2}\right)+9$

$x=\frac{2t+1}{t-2} \to f(x)=\frac{17x-11}{x-2}$

ب) $f(x+\frac{1}{x})=x^2+\frac{1}{x^2}$ (۹)

$f(x)=?$ $\quad (x+\frac{1}{x})=t$

$t^2=x^2+\frac{1}{x^2}+2\underbrace{(x+\frac{1}{x})}_{t}$ (۶)

$t^2=x^2+\frac{1}{x^2}+2t$

$x^2+\frac{1}{x^2}=t^2-2t$

$f(t)=t^2-2t$

$f(x)=x^2-2x$

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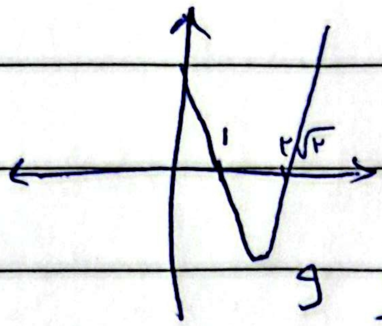
$f(x) = x\sqrt{x}$ $\quad x \geq 0$

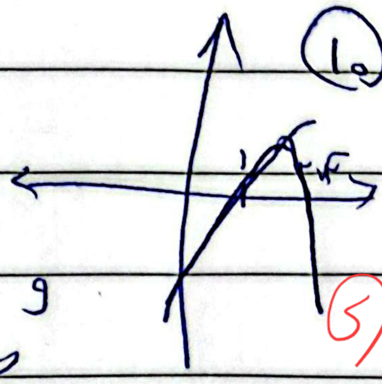
$g(f(x)) = 0$

$g(x) = 0$

$\hookrightarrow x_1 = 1$

$x_2 = ۲\sqrt{۲}$





$$f(x) = \begin{cases} x_1 = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x_1' = 1 \\ x_2 = ۲\sqrt{۲} \rightarrow x\sqrt{x} = ۲\sqrt{۲} \rightarrow x_2' = ۲ \end{cases}$$

$x_2' - x_1' = ۲ - 1 = 1$