

بار دوم دفتر

تکلیف ۹

بانیس سناه نظری

$$f(n) = \begin{cases} \cot \frac{\pi n}{r} & ; n \leq 1 \\ \sqrt{n^2 + 1} & ; n > 1 \end{cases}$$

①

$$f \circ f\left(\frac{r}{r}\right) = \quad n = \frac{r}{r} \quad n \leq 1, \cot \frac{\pi \times r}{r \times r} = \cot \frac{\pi}{r} = \sqrt{r}$$

$$n = \sqrt{r} \quad n > 1 \Rightarrow \sqrt{n^2 + 1} = \underline{r}$$

$$\text{الف) } f\left(\frac{1}{n}\right) = \sqrt{\frac{r n - 1}{n^2}}$$

②

$$g(n) = r \cos^n n \rightarrow g\left(\frac{\pi}{r}\right) = r \cos^r\left(\frac{\pi}{r}\right) = \frac{1}{r}$$

$$f \circ g\left(\frac{\pi}{r}\right) = \rightarrow f\left(\frac{1}{r}\right) = \sqrt{\frac{r \times r - 1}{r}} = \boxed{\frac{\sqrt{r}}{r}}$$

$$\text{ب) } f(n) = [n]$$

$$g(n) = \frac{n}{1-n} \rightarrow g(\sqrt{r}) = \frac{\sqrt{r}(1+\sqrt{r})}{1-\sqrt{r}(1+\sqrt{r})} = \frac{r+\sqrt{r}}{-1} = -r-\sqrt{r}$$

$$f \circ g(\sqrt{r}) =$$

$$f(-r-\sqrt{r}) = [-r-\sqrt{r}] = [-\sqrt{r}] - r = \boxed{-r}$$

$$f(n) = \sin n \rightarrow f\left(\frac{\pi}{r}\right) = \sin\left(\frac{\pi}{r}\right) = \frac{\sqrt{r}}{r}$$

③

$$g(n) = n \sqrt{1-n^2} \rightarrow g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \sqrt{\frac{1}{r}} = \boxed{\frac{1}{r}}$$

$$g \circ f\left(\frac{\pi}{r}\right) = \underline{\frac{1}{r}}$$

$$f(n) = \{(f, \Delta), (f, \Delta), (\Delta, 1), (\Delta, 1)\}$$

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$$g(n) = \{(f, f), (f, f), (f, \Delta), (\Delta, 1)\}$$

$$a) f \circ g(n) = \{(f, \Delta), (f, \Delta), (f, 1), (\Delta, 1)\}$$

$$n = f \rightarrow g(n) = f \rightarrow f \circ g(f) = \Delta$$

$$n = f \rightarrow g(n) = f \rightarrow f \circ g(f) = \Delta$$

$$n = f \rightarrow g(n) = \Delta \rightarrow f \circ g(f) = 1$$

$$n = \Delta \rightarrow g(n) = 1 \rightarrow f \circ g(\Delta) = 1$$

$$b) g \circ f(n) = \emptyset$$

$$n = f \rightarrow f(n) = \Delta \rightarrow g \circ f(n) = X$$

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$$n = \Delta \rightarrow f(n) = 1 \rightarrow g \circ f(n) = X$$

$$n = 1 \rightarrow f(n) = 1 \rightarrow g \circ f(n) = X$$

$$c) f \circ f(n) = \emptyset$$

$$n = f \rightarrow f(n) = \Delta \rightarrow f \circ f(n) = X$$

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$$n = \Delta \rightarrow f(n) = 1 \rightarrow f \circ f(n) = X$$

$$n = 1 \rightarrow f(n) = 1 \rightarrow f \circ f(n) = X$$

$$d) g \circ g(n) = \{(f, \Delta), (f, \Delta), (f, 1)\}$$

$$n = f \rightarrow g(n) = f \rightarrow g \circ g(n) = \Delta$$

$$n = f \rightarrow g(n) = f \rightarrow g \circ g(n) = \Delta$$

$$n = f \rightarrow g(n) = \Delta \rightarrow g \circ g(n) = 1$$

$$n = \Delta \rightarrow g(n) = 1 \rightarrow g \circ g(n) = X$$



$$f(n) = \{(r, 1), (r, r), (r, \omega), (1, v)\}$$

$$g(n) = \{(1, r), (r, 1), (a, r), (b, 1)\}$$

$$(r, r) \in f \circ g(n) \rightarrow n = r \rightarrow g(r) = m \rightarrow f(m) = r$$

$$(r, 1) \in g \circ f(n) \rightarrow n = r \rightarrow f(r) = \omega \rightarrow g(\omega) = 1$$

$$(a, b) \in ? \rightarrow \boxed{(r, \omega)} \quad \boxed{a = r} \quad \boxed{b = \omega}$$

$$f \circ f(n) = rn + r \rightarrow f(n) = an + b \rightarrow a(an + b) + b = rn + r$$

$$g(rn + r) = rn - r \quad a^2n + ab + b = rn + r$$

$$rn + r = t \rightarrow n = \frac{t - r}{r} \rightarrow g(t) = r\left(\frac{t - r}{r}\right) - r$$

$$g(n) = \frac{rn - r - r}{r} = \frac{rn - 1r}{r}$$

$$a^2 = r \rightarrow a = +r \rightarrow rb = r \rightarrow b = 1$$

$$a = -r \rightarrow -b = r \rightarrow b = -r$$

$$\textcircled{1} f(n) = rn + 1$$

$$\textcircled{2} f(n) = -rn - r$$

$$g \circ f(-1) = \textcircled{1} f(-1) = r(-1) + 1 = -1$$

$$\textcircled{2} f(-1) = -r(-1) - r = -1$$

$$g(-1) = \frac{r(-1) - 1r}{r}$$

$$\boxed{g(f(n)) = -n}$$

$$f(n) = \sqrt{n + |n|} \rightarrow D_f \Rightarrow \begin{matrix} n + |n| \geq 0 \\ n < 0 & n = 0 & n > 0 \end{matrix}$$

$$g(n) = \frac{1}{n^2 - rn}$$

$$D_{f \circ g} = D_f \cap D_g$$

$$f(n) = 0 \rightarrow D_f = \mathbb{R}$$

$$f(n) = \sqrt{rn} \rightarrow D_f = \mathbb{R}$$

$$f(n) = 0 \rightarrow D_f = \mathbb{R}$$

$$D_f = D_{f_1} \cup D_{f_2} \cup D_{f_3} = \mathbb{R}$$



$$g(x) = \frac{1}{x^r - r} \rightarrow Dg = x^r - r \neq 0$$

$$(x)(x-r) \neq 0 \rightarrow \begin{matrix} x \neq 0 \\ x \neq r \end{matrix}$$

$$Dg = \mathbb{R} - \{0, r\}$$

$$D_{g \circ f} = D_f \cap D_g = \mathbb{R} - \{0, r\}$$

$$f(x) = \sqrt{1-x^2} \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \quad \textcircled{A}$$

$$g(x) = \sqrt{x} \rightarrow D_g = [0, \infty)$$

$$f(x) + g(x) = \sqrt{x} + \sqrt{1-x^2} \rightarrow \begin{matrix} 1-x^2 \geq 0 \rightarrow x \in [-1, 1] \\ x \geq 0 \rightarrow x \in [0, \infty) \end{matrix} \rightarrow D_{f+g} = [0, 1]$$

$$D_{(f+g) \circ f} = D_f \cap D_{(f+g)} = [0, 1]$$

$$1) f\left(\frac{r_{n+1}}{n-r}\right) = r_{n+1} \rightarrow \frac{r_{n+1}}{n-r} = t \quad \textcircled{7}$$

$$r_{n+1} = nt - r$$

$$(r-t)n = -1 - rt$$

$$f(t) = \frac{r}{\left(\frac{rt+1}{t-r}\right) + \omega} \leftarrow n = \frac{rt+1}{t-r} \leftarrow n = \frac{-1-rt}{r-t}$$

$$f(t) = \frac{1t + r + \omega t - \omega}{t-r} = \frac{1t - 1}{t-r} \rightarrow \boxed{f(x) = \frac{1x - 1}{x-r}}$$

$$\rightarrow f\left(x + \frac{1}{n}\right) = x^r + \frac{1}{n^r} \rightarrow x^r + \frac{1}{n^r} = \left(x + \frac{1}{n}\right) \left(x^r + \frac{1}{n^r} - 1\right)$$

$$\left(x + \frac{1}{n}\right)^r = x^r + \frac{1}{n^r} + r \quad \textcircled{1} \quad f(x) = \left(x + \frac{1}{n}\right) \left(\left(x + \frac{1}{n}\right)^r - r\right)$$

$$t = x + \frac{1}{n} \rightarrow f(t) = t(t^r - r) \rightarrow \boxed{f(x) = x(x^r - r)}$$

$$\{(1\sqrt{2}, 0), (1, 0)\} \in g(n)$$

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$$g \circ f(n) = 0 \rightarrow n = a \rightarrow f(n) = a\sqrt{a} \rightarrow g(a\sqrt{a}) = 0$$

$$\begin{cases} a\sqrt{a} = 1 \rightarrow n = a_1 = 1 \\ a\sqrt{a} = 1\sqrt{2} \rightarrow n = a_2 = 2 \end{cases}$$

$$a_2 - a_1 = 2 - 1 = \boxed{1}$$