

$$1) f\left(\frac{x}{y}\right) = \cot \frac{x}{y} \quad \pi = \cot \frac{\pi}{4}, \sqrt{x} \quad f \circ f\left(\frac{x}{y}\right) = y$$

$$f(\sqrt{x}) = \sqrt{(\sqrt{x})^2 + 1} = \sqrt{x+1}$$

$$2) \text{ ا) } g\left(\frac{x}{y}\right) = y\left(\frac{x}{y}\right)^y, \frac{1}{y} \rightarrow \frac{1}{x} = \frac{1}{y} \rightarrow x = y \rightarrow f(y) = y$$

$$\frac{\sqrt{xy-1}}{x} = \frac{\sqrt{y}}{y} \quad f \circ g\left(\frac{x}{y}\right) = \frac{\sqrt{y}}{y}$$

$$b) g(\sqrt{x}) = \frac{\sqrt{x}}{1-\sqrt{x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{\sqrt{x}+x}{-1} = -x-\sqrt{x} \rightarrow f(-x-\sqrt{x}) = [-x-\sqrt{x}]$$

$$f \circ g(\sqrt{x}) = -x$$

$$3) f\left(\frac{x}{y}\right) = \sin \frac{x}{y}, \frac{\sqrt{x}}{y} \quad g\left(\frac{\sqrt{x}}{y}\right) = \frac{\sqrt{x}}{y} \sqrt{1 - \left(\frac{\sqrt{x}}{y}\right)^2}, \frac{\sqrt{x}}{y} \times \frac{1}{\sqrt{x}} = \frac{1}{y}$$

$$g \circ f\left(\frac{x}{y}\right) = \frac{1}{y}$$

$$4) f \circ g(x) = [(x, \omega), (y, \omega), (x, y), (y, \omega)]$$

$$g \circ f(x) = \emptyset \quad f \circ f(x) = \emptyset$$

$$g \circ g(x) = [(x, 1), (y, y), (y, 1)]$$

$$5) a \rightarrow y \rightarrow x$$

$$(a, y) = (x, y) \Rightarrow a = x$$

$$x \rightarrow \omega \rightarrow 1$$

$$(b, 1) = (\omega, 1) \rightarrow b = \omega$$

$$(a, b) = (x, \omega)$$

$$6) f(f(a)) = a(a+1)+1 = ax+x$$

$$\frac{a^2x+b(a+1)}{x} = \frac{ax+x}{x} \rightarrow a = x$$

$$g(x+y) = x^2-1 \quad \frac{x^2+y^2}{x^2-1} = \frac{y^2-1}{x^2-1}$$

$$g \circ f(-1) = \frac{1}{-1} = -1 \quad g(-1) = \frac{-1}{-1} = 1$$

$$g \circ f(-1) = -1 \quad g(-1) = 1$$

$$7) f(x) = \sqrt{x+1} \rightarrow D_{g \circ f} = \{x \in D_f, f(x) \in D_g\}$$

$$g(x) = \frac{1}{x^2-x} \rightarrow x \neq 0, x \neq 1$$

$$\sqrt{x+1} \neq 0 \rightarrow x > -1$$

$$\sqrt{x+1} \neq 1 \rightarrow x \neq 0$$

$$D_{g \circ f} = (-1, +\infty) - \{0\}$$

$$8) (f+g) \circ f \rightarrow [0, 1] \cup (\sqrt{1-x^2}) = [-1, 1]$$

$$\sqrt{1-x^2} + \sqrt{x} \rightarrow [0, 1]$$

$$-1 \leq x \leq 1 \quad x_2 \rightarrow [0, 1]$$

$$9) \text{ ا) } \frac{x^2+1}{x-1} = x \rightarrow x^2-1+x = x^2+1$$

$$x(x-1) = x^2+1 \quad x = \frac{x^2+1}{x-1}$$

$$f(x) = \frac{x^2+1}{x-1} = x \rightarrow x^2-1+x = x^2+1$$

$$f(x) = x^2-1+x \rightarrow x^2-1+x = x^2+1$$

$$f(x) = x^2-1+x$$

$$\begin{array}{l}
 10 \left| \begin{array}{l}
 g(f(x)) = 0 \longrightarrow f(x) = 1 \quad \subseteq \quad f(x) = 1\sqrt{x} \\
 \sqrt[16]{16\sqrt{x}} \quad \quad \quad x\sqrt{x} = 1 \quad \quad \quad x\sqrt{x} = 4\sqrt{x} \\
 \quad \quad \quad \quad \quad \quad x = 1 \quad \quad \quad \quad \quad x = 4 \\
 x_2 - x_1 = 4 - 1 = 1
 \end{array} \right.
 \end{array}$$

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