

$$f(u) = \begin{cases} \cot \frac{\pi}{2} & ; u \leq 1 \rightarrow \cot \frac{\frac{\pi}{2}}{2} = \cot \frac{\pi}{4} = \sqrt{3} \\ \sqrt{u^2+1} & ; u > 1 \rightarrow \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2 \end{cases}$$

$$\log\left(\frac{\pi}{2}\right) \Rightarrow f\left(g\left(\frac{\pi}{2}\right)\right) = f(\sqrt{3}) = \underline{(2)} \rightarrow \text{جواب}$$

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$$\text{الف) } f\left(\frac{1}{u}\right) = \sqrt{\frac{u-1}{u^2}} \quad , \quad g(u) = 2\cos^2 u \Rightarrow g\left(\frac{\pi}{4}\right) = 2\cos^2 \frac{\pi}{4} = 2 \times \frac{1}{2} = 1$$

$$\log\left(\frac{\pi}{4}\right) \rightarrow f\left(g\left(\frac{\pi}{4}\right)\right)$$

$$\text{ج) } \rightarrow f\left(\frac{1}{u}\right) = \sqrt{\frac{u-1}{u^2}} \Rightarrow f\left(\frac{1}{2}\right) \rightarrow \sqrt{\frac{2-1}{4}} = \frac{\sqrt{1}}{2} = \frac{1}{2} \rightarrow \text{جواب}$$

$$\text{ب) } f(u) = [u] \quad , \quad g(u) = \frac{u}{1-u} \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -(\sqrt{2}+2)$$

$$\log(\sqrt{2}) \rightarrow f\left(g(\sqrt{2})\right) = \underline{(-2)} \rightarrow \text{جواب}$$

$$\text{د) } \rightarrow f(u) = [u] \rightarrow [-\sqrt{2}-2] = -2$$

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$$f(u) = \sin u \quad , \quad g(u) = u\sqrt{1-u^2} \rightarrow \frac{\sqrt{2}}{2} \sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \underline{\left(\frac{1}{2}\right)} \rightarrow \text{جواب}$$

$$\text{د) } \rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

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$$\text{الف) } f \circ g(u) \rightarrow f(g(u)) = \{(2, \omega), (2, \omega), (4, 12), (8, 12)\} \rightarrow \text{جواب}$$

$$\begin{array}{l} \xrightarrow{g(u)} \xrightarrow{f(u)} \quad \omega \quad 4 \rightarrow 8 \rightarrow 12 \\ 2 \rightarrow 4 \rightarrow \omega \quad 8 \rightarrow 10 \rightarrow 12 \end{array}$$

$$\text{ب) } g \circ f(u) = \emptyset \rightarrow \text{جواب}$$

$$\text{ج) } f \circ f(u) = \emptyset \rightarrow \text{جواب}$$

$$\text{د) } g \circ g(u) = \{(2, 8), (2, 4), (4, 10)\} \rightarrow \text{جواب}$$

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$$f = \{(2, 1), (3, 2), (4, \omega), (1, \nu)\}$$

$$g = \{(1, 2), (2, 1), (a, 3), (b, 1)\}$$

$$(2, 2) \in f \circ g \quad , \quad (2, 1) \in g \circ f$$

$$f(g(u)) \rightarrow \begin{array}{l} \xrightarrow{g(u)} \xrightarrow{f(u)} \quad 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 1 \rightarrow \nu \\ a \rightarrow 3 \rightarrow 2 \\ b \rightarrow 1 \rightarrow \nu \end{array} \Rightarrow \{(1, 1), (3, \nu), (a, 2), (b, \nu)\}$$

$$(2, 2) \in f \circ g$$

$$(a, b) = \underline{(2, \omega)} \rightarrow \text{جواب}$$

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$$f(f(u)) = 2u + 3 \rightarrow 2x - 1 + 3 = -1$$

$$g(2u + 3) = 3u - 2 \rightarrow 2u + 3 = t \rightarrow u = \frac{t-3}{2}$$

$$g \circ f(-1) \rightarrow g(f(-1))$$

$$g(-1) \rightarrow 2u + 3 = -1 \rightarrow 2u = -4 \rightarrow u = -2$$

$$g(-1) = 3x - 2 = (-1) \rightarrow \text{جواب}$$

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$$f(u) = \sqrt{u+|u|} \quad , \quad g(u) = \frac{1}{u^2 - 2u}$$

$$D_{g \circ f} = \{ u \in D_f : f(u) \in D_g \}$$

$$\bullet u \in \mathbb{R} \setminus \{0\} \quad \bullet \sqrt{u+|u|} \in \mathbb{R} \setminus \{0, 2\}$$

$$(I) \cap (II) = (0, 2 + \infty) = \{2\}$$

جواب

$$D_f \rightarrow u+|u| \geq 0 \rightarrow \text{جواب} \rightarrow D_f = \mathbb{R} \quad \bullet$$

$$D_g \rightarrow \frac{1}{u^2 - 2u} \rightarrow u^2 - 2u \neq 0 \rightarrow u \neq 0, u \neq 2 \rightarrow D_g = \mathbb{R} \setminus \{0, 2\} \quad \bullet$$

$$\sqrt{u+|u|} \neq 0 \rightarrow u+|u| \neq 0 \rightarrow u > 0 \quad , \quad \sqrt{u+|u|} \neq 2 \rightarrow u+|u| \neq 4 \rightarrow u \neq 1 \quad (II)$$

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$$f(u) = \sqrt{1-u^2} \quad , \quad g(u) = \sqrt{u}$$

$$(f+g) \circ f \rightarrow f \circ f + g \circ f = \sqrt{1-(1-u^2)} + \sqrt{1-u^2} = \pm u + \sqrt{1-u^2} \Rightarrow 1-u^2 \geq 0 \Rightarrow$$

$$\frac{-1}{-1} \leq \frac{1}{1} \rightarrow D_{(f+g) \circ f} = [-1, 1] \rightarrow \text{جواب}$$

$u=1$
 $u=-1$

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$$\text{ب) } f\left(\frac{3u+1}{u-2}\right) = 2u+5 \rightarrow \frac{3u+1}{u-2} = t \rightarrow 3u+1 = tu-2t \rightarrow u(3-t) = -2t-1 \rightarrow$$

$$u = \frac{2t+1}{t-3} \rightarrow f(t) = \frac{1t+2+5t-15}{t-3} \rightarrow f(t) = \frac{6t-11}{t-3} \rightarrow f(u) = \frac{6u-11}{u-3} \Rightarrow \text{جواب}$$

$u=3$

$$\text{ج) } f\left(u + \frac{1}{u}\right) = u^2 + \frac{1}{u^2} \rightarrow u + \frac{1}{u} = t$$

$$t^2 = u^2 + \frac{1}{u^2} + 2u + \frac{2}{u} \rightarrow t^2 - 2t = u^2 + \frac{1}{u^2} \rightarrow f(t) = t^2 - 2t \rightarrow f(u) = u^2 - 2u$$

$\frac{2(u + \frac{1}{u})}{t}$

جواب

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$$g(u) = (u-1)(u-2\sqrt{2}) \quad , \quad f(u) = u\sqrt{u} \rightarrow g(f(u)) = (u\sqrt{u}-1)(u\sqrt{u}-2\sqrt{2})$$

$$g(f(u)) = (u\sqrt{u}-1)(u\sqrt{u}-2\sqrt{2}) \rightarrow u=1, 2$$

$$12-11 = 1$$

جواب

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