

$$f(u) = \begin{cases} \cot \frac{\pi}{2} & ; u \leq 1 \rightarrow \cot \frac{\frac{\pi}{2}}{2} = \cot \frac{\pi}{4} = \sqrt{3} \\ \sqrt{u^2+1} & ; u > 1 \rightarrow \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2 \end{cases}$$

$$\log\left(\frac{\pi}{2}\right) \Rightarrow f\left(g\left(\frac{\pi}{2}\right)\right) = f(\sqrt{3}) = 2 \rightarrow \text{جواب}$$

$$\text{الف) } f\left(\frac{1}{u}\right) = \sqrt{\frac{u-1}{u^2}} \quad , \quad g(u) = 2\cos^2 u \Rightarrow g\left(\frac{\pi}{4}\right) = 2\cos^2 \frac{\pi}{4} = 2 \times \frac{1}{2} = 1$$

$$\log\left(\frac{\pi}{4}\right) \rightarrow f\left(g\left(\frac{\pi}{4}\right)\right)$$

$$\rightarrow f\left(\frac{1}{1}\right) = \sqrt{\frac{1-1}{1^2}} = 0 \rightarrow \text{جواب}$$

$$\text{ب) } f(u) = [u] \quad , \quad g(u) = \frac{u}{1-u} \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = -(\sqrt{2}+2)$$

$$\log(\sqrt{2}) \rightarrow f\left(g(\sqrt{2})\right) = f(-(\sqrt{2}+2)) = -2 \rightarrow \text{جواب}$$

$$\rightarrow f(u) = [u] \rightarrow [-\sqrt{2}-2] = -2$$

$$f(u) = \sin u \quad , \quad g(u) = u\sqrt{1-u^2} \rightarrow \frac{\sqrt{2}}{4} \sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{4} \times \frac{1}{\sqrt{2}} = \frac{1}{4}$$

$$g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow g\left(\frac{\sqrt{2}}{4}\right) = \frac{1}{4} \rightarrow \text{جواب}$$

$$\rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$\text{الف) } f \circ g(u) \rightarrow f(g(u)) = \{(2, \omega), (2, \omega), (4, 12), (8, 12)\} \rightarrow \text{جواب}$$

$$\begin{matrix} 2 \xrightarrow{g(u)} 4 \xrightarrow{f(u)} \omega & 4 \rightarrow 8 \rightarrow 12 \\ 2 \rightarrow 4 \rightarrow \omega & 8 \rightarrow 16 \rightarrow 12 \end{matrix}$$

$$\text{ب) } g \circ f(u) = \emptyset \rightarrow \text{جواب}$$

$$\text{ج) } f \circ f(u) = \emptyset \rightarrow \text{جواب}$$

$$\text{د) } g \circ g(u) = \{(2, 8), (2, 4), (4, 16)\} \rightarrow \text{جواب}$$

$$f = \{(2, 1), (3, 2), (4, \omega), (1, \nu)\}$$

$$g = \{(1, 2), (2, 1), (a, 3), (b, 1)\}$$

$$(2, 2) \in f \circ g \quad , \quad (2, 1) \in g \circ f$$

$$f(g(u)) \rightarrow \begin{matrix} 1 \xrightarrow{g(u)} 2 \xrightarrow{f(u)} 1 \\ 3 \rightarrow 1 \rightarrow \nu \\ a \rightarrow 2 \rightarrow 1 \\ b \rightarrow 1 \rightarrow \nu \end{matrix} \Rightarrow \{(1, 1), (3, \nu), (a, 1), (b, \nu)\}$$

$$(2, 2) \in f \circ g$$

$$(a, b) = (2, \omega) \rightarrow \text{جواب}$$

$f(f(u)) = 2u+3 \rightarrow 2x-1+3=-1$ $g(2u+3) = 3u-2 \rightarrow 2u+3=t \rightarrow u = \frac{t-3}{2}$ $g \circ f(-1) \rightarrow g(f(-1))$ $g(-1) \rightarrow 2u+3=-1 \rightarrow 2u=-4 \rightarrow u=-2$ $g(-1) = 3x-2-2 = (-1) \rightarrow \text{جواب}$	5 6
$f(u) = \sqrt{u+ u } \quad , \quad g(u) = \frac{1}{u^2-2u}$ $D_{g \circ f} = \{ u \in D_f : f(u) \in D_g \}$ $\bullet u \in \mathbb{R} \setminus \{0\}$ $\bullet \sqrt{u+ u } \in \mathbb{R} \setminus \{0, 2\}$ $D_f \rightarrow u+ u \geq 0 \rightarrow \text{دائماً} \rightarrow D_f = \mathbb{R} \quad \text{جواب}$ $D_g \rightarrow \frac{1}{u^2-2u} \rightarrow u^2-2u \neq 0 \rightarrow u \neq 0, u \neq 2 \rightarrow D_g = \mathbb{R} \setminus \{0, 2\}$ $\sqrt{u+ u } \neq 0 \rightarrow u+ u \neq 0 \rightarrow u > 0 \quad , \quad \sqrt{u+ u } \neq 2 \rightarrow u+ u \neq 4 \rightarrow u \neq 1 \quad (\text{II})$	5 7
$f(u) = \sqrt{1-u^2} \quad , \quad g(u) = \sqrt{u}$ $(f+g) \circ f \rightarrow f \circ f + g \circ f = \sqrt{1-(1-u^2)} + \sqrt{1-u^2} = \pm u + \sqrt{1-u^2} \Rightarrow 1-u^2 \geq 0 \Rightarrow$ $\frac{-1}{-1+1} \rightarrow D_{(f+g) \circ f} = [-1, 1] \rightarrow \text{جواب}$ $u=1$ $u=-1$	5 8
ب) $f\left(\frac{3u+1}{u-2}\right) = 2u+5 \rightarrow \frac{3u+1}{u-2} = t \rightarrow 3u+1 = tu-2t \rightarrow u(3-t) = -2t-1 \rightarrow$ $u = \frac{2t+1}{t-3} \rightarrow f(t) = \frac{1t+2+5t-15}{t-3} \rightarrow f(t) = \frac{6t-11}{t-3} \rightarrow f(u) = \frac{6u-11}{u-3} \Rightarrow \text{جواب}$ $u=3$ ج) $f\left(u + \frac{1}{u}\right) = u^2 + \frac{1}{u^2} \rightarrow u + \frac{1}{u} = t$ $t^2 = u^2 + \frac{1}{u^2} + 2u + \frac{2}{u} \rightarrow t^2 - 2t = u^2 + \frac{1}{u^2} \rightarrow f(t) = t^2 - 2t \rightarrow f(u) = u^2 - 2u$ $2(u + \frac{1}{u})$	5 9
$g(u) = (u-1)(u-2\sqrt{2}) \quad , \quad f(u) = u\sqrt{u} \Rightarrow g(f(u)) = (u\sqrt{u}-1)(u\sqrt{u}-2\sqrt{2})$ $g(f(u)) = (u\sqrt{u}-1)(u\sqrt{u}-2\sqrt{2}) \rightarrow u=1, 2$ $12-11 = 1 \rightarrow \text{جواب}$	5 10