

$$f(x) = \begin{cases} \cot \frac{\pi x}{\pi} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \quad f \circ f\left(\frac{y}{\pi}\right) : f\left(f\left(\frac{y}{\pi}\right)\right) = f(\sqrt{\pi}) = \sqrt{(\sqrt{\pi})^2 + 1} = 2 \quad -1$$

$$f\left(\frac{y}{\pi}\right) = \cot \frac{\pi}{\pi} \times \frac{y}{\pi} = \cot \frac{\pi}{y} = \sqrt{\pi} \quad -2$$

$$\text{الف) } f \circ g\left(\frac{\pi}{\pi}\right) = f\left(g\left(\frac{\pi}{\pi}\right)\right) = f\left(\frac{1}{\pi}\right) = \sqrt{\frac{\pi-1}{\pi}} = \sqrt{\frac{\pi}{\pi}} = \sqrt{\frac{\pi}{\pi}} \quad -2$$

$$g\left(\frac{\pi}{\pi}\right) = \pi \cos^{\pi} \frac{\pi}{\pi} = \pi \times \left(\frac{1}{\pi}\right)^{\pi} = \frac{1}{\pi}$$

$$\text{ب) } f \circ g(\sqrt{\pi}) = f\left(g(\sqrt{\pi})\right) = f(-\sqrt{\pi} - \pi) = [-\sqrt{\pi} - \pi] = \underbrace{[-\sqrt{\pi}]}_{\sqrt{\pi} \approx 1.77} - \pi = -1.77 - \pi = -\pi \quad -2$$

$$g(\sqrt{\pi}) = \frac{\sqrt{\pi}}{1-\sqrt{\pi}} \times \frac{1+\sqrt{\pi}}{1+\sqrt{\pi}} = \frac{\sqrt{\pi} + \pi}{-1} = -\sqrt{\pi} - \pi$$

$$g \circ f\left(\frac{\pi}{\pi}\right) = g\left(f\left(\frac{\pi}{\pi}\right)\right) = g\left(\frac{\sqrt{\pi}}{\pi}\right) = \frac{\sqrt{\pi}}{\pi} \times \sqrt{1 - \left(\frac{\sqrt{\pi}}{\pi}\right)^2} = \frac{\sqrt{\pi}}{\pi} \times \sqrt{\frac{1}{\pi}} = \frac{1}{\pi} \quad -3$$

$$f\left(\frac{\pi}{\pi}\right) = \sin \frac{\pi}{\pi} = \frac{\sqrt{\pi}}{\pi} \quad -4$$

$$f(x) = \{(x, \omega)(y, \omega)(\pi, \pi)(10, \pi)\} \quad g(x) = \{(x, y)(y, x)(y, \pi)(\pi, 1)\}$$

$$\text{الف) } f \circ g(x) = \{(x, \omega)(y, \omega)(y, \pi)(\pi, \pi)\}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

$$\Rightarrow g \circ g(x) = \{(x, \pi)(y, y)(y, 1)\}$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$f(x) = \{(x, 1)(\pi, \pi)(\pi, \omega)(1, \pi)\} \quad g(x) = \{(1, \pi)(\pi, 1)(a, \pi)(b, 1)\} \quad -5$$

$$(x, \pi) \in f \circ g \quad (x, 1) \in g \circ f \rightarrow g(f(x)) = 1 \xrightarrow{f(x)=\omega} g(\omega) = 1 \rightarrow b = \omega$$

$$\hookrightarrow f(g(x)) = \pi \Rightarrow g(x) = \pi \rightarrow a = \pi$$

$$(a, b) = (\pi, \omega)$$

$$f(f(x)) = \pi x + \pi \quad g(\pi x + \pi) = \pi x - \pi \quad g \circ f(-1) = ? \quad -6$$

$$f(x) = ax + b \rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = \pi x + \pi$$

$$\Rightarrow a^2 = \pi \rightarrow a = \pi \rightarrow \pi b + b = \pi b = \pi \Rightarrow b = 1 \xrightarrow{1} f(x) = \pi x + 1$$

$$\hookrightarrow a = -\pi \rightarrow -\pi b + b = -b = \pi \Rightarrow b = -\pi \xrightarrow{2} f(x) = -\pi x - \pi$$

$$\pi x + \pi = t \Rightarrow \pi x = t - \pi \Rightarrow x = \frac{t - \pi}{\pi} \Rightarrow g(t) = \pi x \frac{t - \pi}{\pi} - \pi = \frac{\pi t - \pi - \pi}{\pi}$$

$$= \frac{10}{7}t - \frac{10}{7}$$

$$\textcircled{1} \text{ جواب : } g \circ f(-1) = g(f(-1)) = g(-1) = \frac{10}{7} \times (-1) - \frac{10}{7} = -\frac{10}{7} - \frac{10}{7} = -\frac{20}{7}$$

$$f(-1) = -2 + 1 = -1$$

$$\textcircled{2} \text{ جواب : } g \circ f(-1) = g(f(-1)) = g(-1) = -\frac{20}{7}$$

$$f(-1) = -2 \times (-1) - 1 = -1$$

$$g \circ f(x) \begin{cases} x \in D_f \rightarrow x+|x| \geq 0 \xrightarrow{x > 0} x \geq 0 \\ f \in D_g \xrightarrow{x=0} 0=0 \\ \xrightarrow{x < 0} -x+x \geq 0 \end{cases} \quad D_f = \mathbb{R}$$

$$x^2 - \varepsilon x \neq 0$$

$$x \neq 0, x \neq \varepsilon$$

$$\sqrt{x+|x|} \neq 0 \Rightarrow x+|x| \neq 0 \Rightarrow x > 0$$

$$\sqrt{x+|x|} \neq \varepsilon \Rightarrow x+|x| \neq \varepsilon^2 \Rightarrow x \neq \varepsilon^2$$

$$(f+g) \circ f = f(f(x)) + g(f(x)) = \sqrt{1-(\sqrt{1-x^2})^2} + \sqrt{1-x^2} =$$

$$1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$$

$$1-(\sqrt{1-x^2})^2 \geq 0 \Rightarrow (\sqrt{1-x^2})^2 \leq 1 \Rightarrow \underbrace{-1 \leq \sqrt{1-x^2} \leq 1}_{\text{Giv}} \Rightarrow 1-x^2 \leq 1 \Rightarrow$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$\text{الف) } f\left(\frac{10x+1}{x-2}\right) = \varepsilon x + \omega \quad \frac{10x+1}{x-2} = t \Rightarrow tx - 2t = 10x + 1 \Rightarrow (t-10)x = 2t+1 \Rightarrow x = \frac{2t+1}{t-10}$$

$$f(t) = 10 \times \frac{2t+1}{t-10} + \omega = \frac{20t+10+\omega t-10\omega}{t-10} = \frac{13t-11}{t-10} \quad f(x) = \frac{13x-11}{x-10}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \Rightarrow f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) \Rightarrow x^3 + \frac{1}{x^3} = t^3 - 3t$$

$$g(1) = 0, g(2\sqrt{2}) = 0 \quad f(x) = x\sqrt{x}$$

$$g \circ f(x) = g(f(x)) = 0 \quad \begin{cases} f(x) = 1 \Rightarrow x = 1 \\ f(x) = 2\sqrt{2} \Rightarrow x = 2 \end{cases} \quad 2-1=1$$

$$\text{!} \quad \text{مراجعة سريعة}$$