یازدهم ریاضی C — تکلیف - ۹

Date No

1- $f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$ $\quad f\circ f(\frac{2}{3})=f(\cot\frac{\pi}{6})=\sqrt{(\sqrt{3})^2+1}=2$

2- الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $\quad g(x)=2\cos^2 x$ $\quad f(g(\frac{\pi}{3}))=f(\underbrace{2\cos^2\frac{\pi}{3}}_{\frac{1}{4}\times 2})=f(\frac{1}{2})=0$

ب) $f(x)=[x]$ $\quad g(x)=\frac{x}{1-x}$ $\quad f(g(\sqrt{2}))=f(\frac{\sqrt{2}}{1-\sqrt{2}})=[\frac{\sqrt{2}}{1-\sqrt{2}}]$

$\rightarrow \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{1-2}=-\underbrace{\sqrt{2}}_{1,\dots}-2=-3,\dots \Rightarrow [-3,\dots]=-4 \Rightarrow f\circ g(\sqrt{2})=-4$

3- $f(x)=\sin x$ $\quad g(f(\frac{\pi}{4}))=g(\sin\frac{\pi}{4})=g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$

$g(x)=x\sqrt{1-x^2}$

4- $f(x)=\{(4,5)(6,5)(8,13)(10,12)\}$

$g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

الف) $f\circ g(x)=\{(4,5)(2,5)(6,12)(8,12)\}$

ب) $g\circ f(x)=\varnothing$

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5- $f=\{(2,1)(3,2)(4,5)(1,7)\}$ $\quad (4,2)\in f\circ g \rightarrow f(g(4))=2\rightarrow f(3)=2\Rightarrow g(4)=3$

$g=\{(1,2)(3,1)(a,3)(b,1)\}$ $\quad (4,1)\in g\circ f\rightarrow g(f(4))=1\rightarrow g(5)=1\Rightarrow b=5$ $\Rightarrow a=4$

$\rightarrow \begin{rcases} b=5\\ a=4\end{rcases}\Rightarrow (a,b)=(4,5)\in f(x)$

6- $f(f(x))=4x+3$ $\quad g(2x+3)=3x-2$

$\rightarrow f(x)=ax+b\rightarrow f(ax+b)=4x+3 \Longrightarrow a(ax+b)+b=4x+3\rightarrow a^2x+ab+b=4x+3\rightarrow a=\pm 2,$

$\begin{cases} a=2\rightarrow 2b+b=3\rightarrow b=1 \longrightarrow f(x)=2x+1\\ a=-2\rightarrow -2b+b=3\rightarrow b=-3\longrightarrow f(x)=-2x-3\end{cases}\Bigg\}\ f(-1)=-1$

$g(x)=cx+d\rightarrow g(2x+3)=c(2x+3)+d=2cx+3c+d=3x-2\Rightarrow c=\frac{3}{2},\ d=-\frac{13}{2}$

$\rightarrow g(x)=\frac{3}{2}x-\frac{13}{2}$

$g\circ f(-1)=g(f(-1))=g(-1)=-8$

7- $f(x)=\sqrt{x+|x|}$ $\quad g(x)=\frac{1}{x^2-4x}$

$x+|x|\geq 0 \Rightarrow x\geq 0$

$D_{g\circ f}=\{x\in D_f : f(x)\in D_g\} \longrightarrow D_{g\circ f}=\{x\geq 0 : \sqrt{x+|x|}\neq\{0,4\}\} \Rightarrow D_{g\circ f}=(0,+\infty)-\{1\}$

$x\neq 0 \;,\; x+|x|\neq 16 \Rightarrow x\neq 1$

8- $f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad (f+g)\circ f \Rightarrow f+g=\sqrt{x}+\sqrt{1-x^2}$

$\longrightarrow (f+g)(f(x))=(f+g)(\sqrt{1-x^2})=\sqrt[4]{1-x^2}+\sqrt{1-1+x^2}=\sqrt[4]{1-x^2}+|x| \Rightarrow 1-x^2\geq 0 \rightarrow 1\geq x^2 \Rightarrow 1\geq x\geq -1$

$\Rightarrow D_{(f+g)\circ f}=[-1,1]$

9- الف) $f\left(\frac{2x+1}{x-2}\right)=4x+a \longrightarrow t=\frac{2x+1}{x-2} \Rightarrow x=\frac{2t+1}{t-2} \longrightarrow f(t)=4\left(\frac{2t+1}{t-2}\right)+a \Rightarrow f(x)=\frac{11+3x}{2-x}$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \rightarrow t=x+\frac{1}{x} \rightarrow \left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \Rightarrow f(t)=t^3-3t \Rightarrow f(x)=x^3-3x$

10- $f(x)=x\sqrt{x}$ $\quad g\circ f(x)$ $\quad g(1)=g(2\sqrt{2})=0$

$g\circ f(x)=g(f(x))=g(x\sqrt{x})=0 \longrightarrow$

$g(1)=0 \rightarrow 1=x\sqrt{x} \Rightarrow x=1$

$g(2\sqrt{2})=0 \rightarrow 2\sqrt{2}=x\sqrt{x} \Rightarrow x=2$

$\Rightarrow |x_2-x_1|=1$